PAPER No. 27

Modern Developments in Design and Construction of Filters

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WHAT a Filter Does.—If an electric filter were strictly analogous to a mechanical filter, it might be supposed that it would permit the passage of small currents and obstruct large ones. The electric device which most nearly approaches this is no doubt the circuit-breaker. The electric filter filters out not currents of different strengths but currents of different frequencies. It distinguishes between two electric currents only if they are alternating currents and differ in frequency. For this reason electric filters are more accurately known as electric wave filters.

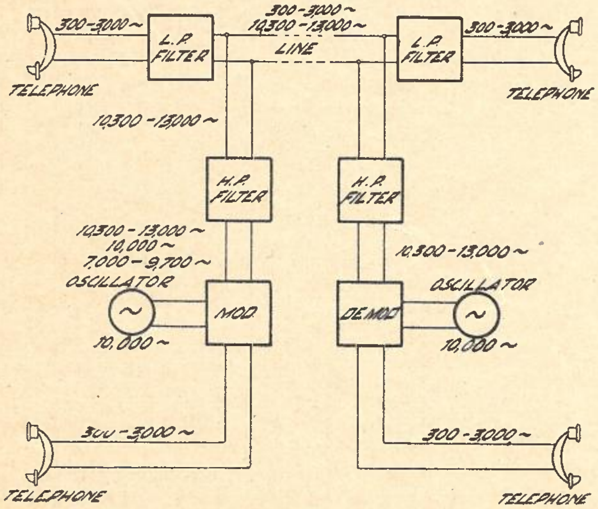


Fig. 1. How Filters are used to derive a Single-channel Carrier Telephone Circuit

There is only one type of mechanical filter or sieve, the kind which lets the smaller particles through and obstructs the larger ones. There are four main types of electric wave filter, however, viz., Low-pass, High-pass, Band-pass and Band-stop. As their names indicate, the first three types pass low frequencies, high frequencies, and a band of frequencies, respectively, while the fourth type passes all but one band of frequencies. There is a further distinction between the action of electric filters and that of sieves; when particles are too big to go through a sieve none go through, but when a current having a frequency in the stop band of an electric filter arrives at the filter it passes through in an attenuated condition. The attenuation it suffers may be anything from 1/100,000 to say 1/10.

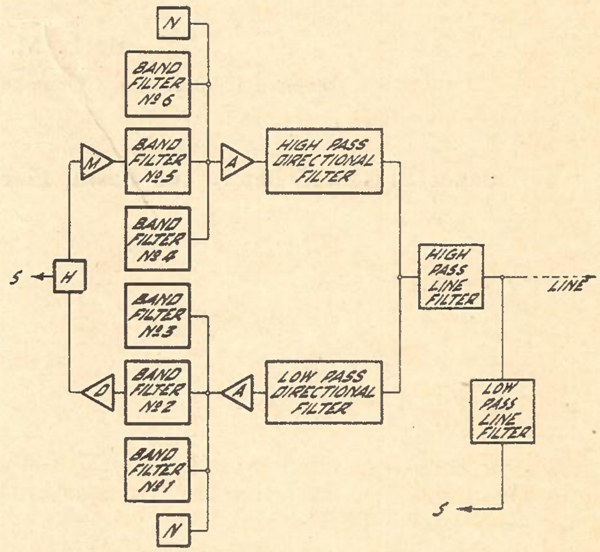


Fig. 2. Filter Arrangements at the Terminal of an Actual Three-channel Carrier Telephone System

Where Filters are Used.—Filters are used wherever it is desired to distinguish between currents of different frequency. There are naturally many applications in postal radio and telephone plant, but the outstanding application is in carrier telephony and telegraphy. As carrier systems are of relatively greater importance in this country than in most others, filter equipment is naturally most important to us.

Fig. 1 shows the part filters play in deriving carrier channels. The line is used to transmit a telephone message of the usual frequency band, and also carries another telephone message occupying the band 10,300 cycles to 13,000 cycles. This latter is the carrier channel, and it is only possible to transmit it over the same pair of wires as the other message without interference, by making use of filters at the ends of the circuit to sort out the two frequency bands. Stepping up the frequency band of the second telephone message from 300-3,000 cycles to 10,300-13,000 cycles is the function of the carrier equipment, and Fig. 1 shows in a rough way how this is done. At the receiving end the demodulator and oscillator bring the frequencies back again to the range 300-3,000 cycles.

Fig. 1 is a hypothetical case, but Fig. 2 shows the arrangement of filters at the terminal of an actual three-channel carrier system. The symbols have the following significance:—

S, line to subscriber;
M, modulator and oscillator;
D, demodulator and oscillator;
H, hybrid coil;
N, compensating network for band-pass group of filters;
A, amplifier.

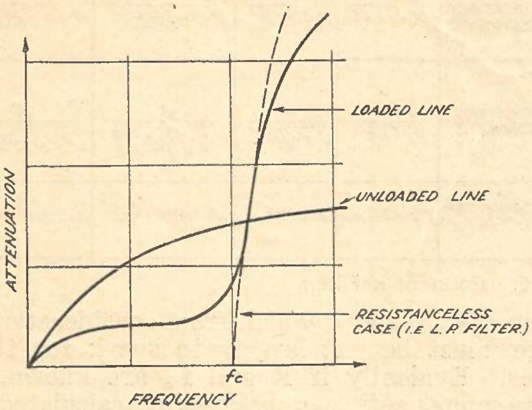


Fig. 3. Relation between Low-pass Filter and the Loaded Line

It will be seen that the line carries an ordinary telephone channel as well as the three-channel carrier system. There are of course three groups of modulator-hybrid-demodulator sets, but only one is shown in the diagram. The filters are the most important part of a carrier system, more important even than the valves, and a carrier system without tubes is within the realm of practical possibilities.

Physical Form of Filters.—Filters are electrical networks in which the elements are inductances and condensers. They are usually mounted on a steel panel for rack mounting and have a soldered-on cover for shielding.

Components.—The condensers are usually mica and are normally quite free of troubles. Sometimes they are clamped to prevent any change in capacity. The inductances are the main difficulty in filter construction. Their effective resistance has to be kept down in order to obtain sharp cut-off in the filter, and special methods of construction have to be adopted to secure this. The commonest form is that wound on an iron-dust or permalloy-dust core of the doughnut shape. Since in these cores the iron particles are very small and are insulated from each other, the eddy current losses are very much reduced, and so the effective resistance of the coil is brought down. For high frequencies and small values of inductance, ordinary solenoid coils with air-cores are sometimes used. The difficulty with these coils is that they have a large external field and have to be shielded. Other precautions necessary in the construction of filters concern the stability of the components, the balance of the network to earth, insulation resistance and the like.

Zobel Filters

In this section an outline, necessarily brief, is given of the Zobel filter. By the Zobel filter is meant the ladder type of filter developed and used by the Bell Telephone System, and it is so called because the name of O. J. Zobel has been most prominently associated with its development.

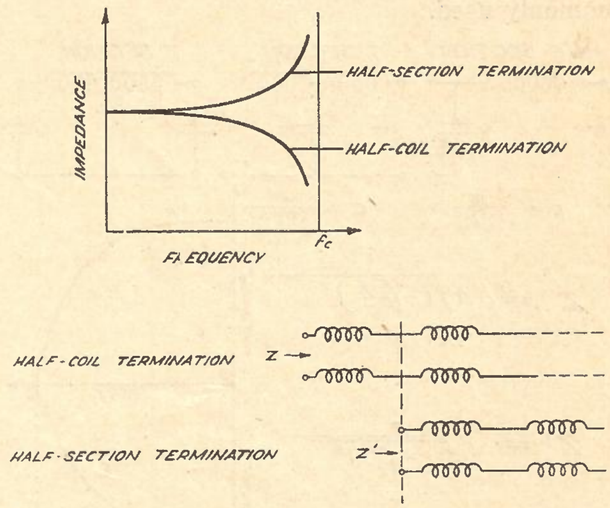


Fig. 4. Impedance Characteristic of the Loaded Line

Origin.—This type of filter originated in a study of the properties of the loaded line by G. A. Campbell of the Bell System. The original patent on the subject was issued to him in 1917. Fig. 3 shows how the attenuation characteristic of the loaded line compares with that of the unloaded line. For the lower frequencies the attenuation is less than that of the unloaded line, but at a certain frequency the attenuation increases suddenly, and beyond this frequency the loaded line has a much greater attenuation than the unloaded line. Campbell found that this cut-off effect was due to the interaction of the inductance of the loading coil and the capacity of the section of line, and that the cut-off frequency was given by

$$f^{c} = 1/\pi\sqrt{LC}$$

where L and C are the values of these two elements.

If the problem be reduced to its simplest terms by using a condenser of capacity C to represent the section of line, and an inductance L of small resistance in place of the loading coil, a low-pass filter is obtained with an attenuation characteristic as given in the third curve of Fig. 3. The loaded line, when its capacity is lumped and its resistances removed, thus leads to the low-pass filter.

Before leaving the loaded line for filters proper, Fig. 4 notes the other characteristic of the line which is affected by loading, viz., the impedance characteristic. This is also similar to the same characteristic of the low-pass filter.

Low-pass Filter Constant-k Type.—The type of filter spoken of above is known as the constant-k type. The name arises simply through the use of the constant k to represent the characteristic impedance of the circuit in which the filter works. Nowadays R and Z are the symbols most commonly used.

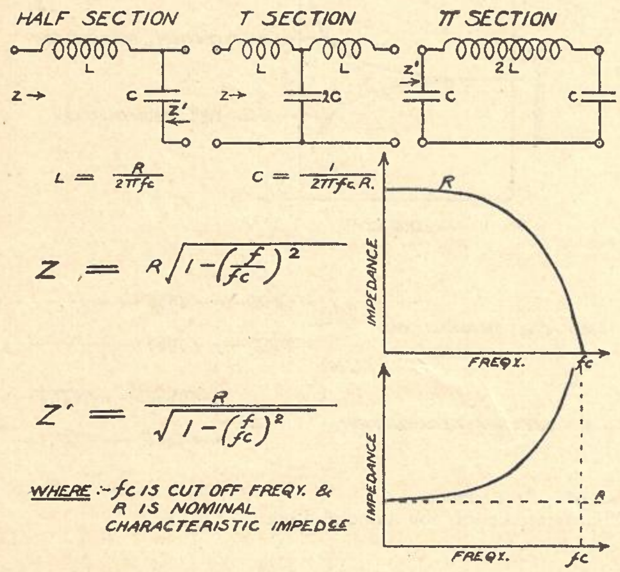


Fig. 5. Properties of the Low-pass Filter, Constant-k Type

The properties of the low-pass filter of the constant-k type are summarised in Fig. 5. In this figure L and C refer to the elements in the half-section, and in this case the cut-off frequency,

$$f = 1/2\pi\sqrt{LC}$$

The figure shows how T and TT sections sections are built up from the half section, and how the impedances of these two symmetrical types of sections are related to the two impedances of the half-section. Formulæ for L and C are given in terms of R and f_c, so that if characteristic impedance and cut-off frequency are known, the filter can be readily worked out. The attenuation of the filter section is that given in Fig. 3.

Summary of all Types of Constant-k Filters.—As well as low-pass filters, there are high-pass, band pass, and band stop filters of the constant-k family. Their configuration and design data are set

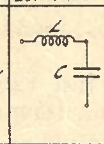
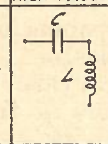
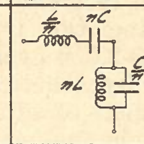
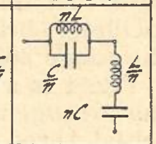
FILTER	LOW PASS	HIGH PASS	BAND PASS	BAND STOP
HALF SECTION				
L	$\frac{R}{2\pi f_c}$	$\frac{R}{2\pi f_c}$	$\frac{R}{2\pi f_m}$	$\frac{R}{2\pi f_m}$
C	$\frac{1}{2\pi f_c R}$	$\frac{1}{2\pi f_c R}$	$\frac{1}{2\pi f_m R}$	$\frac{1}{2\pi f_m R}$
MID-SERIES IMPEDANCE, Z.	$R\sqrt{1-x^2}$	$R\sqrt{1-(\frac{1}{x})^2}$	$R\sqrt{1-\left[\frac{1-x^2}{nx}\right]^2}$	$R\sqrt{1-\left[\frac{nx}{1-x^2}\right]^2}$
MID-SHUNT IMPEDANCE, Z'	$\frac{R}{\sqrt{1-x^2}}$	$\frac{R}{\sqrt{1-(\frac{1}{x})^2}}$	$\frac{R}{\sqrt{1-\left[\frac{1-x^2}{nx}\right]^2}}$	$\frac{R}{\sqrt{1-\left[\frac{nx}{1-x^2}\right]^2}}$
PROPAGATION CONSTANT, θ	$2\,\text{arsinh}\,x$	$2\,\text{arsinh}\,\frac{1}{x}$	$2\,\text{arsinh}\,\frac{1-x^2}{nx}$	$2\,\text{arsinh}\,\frac{nx}{1-x^2}$

Fig. 6. Constant-k Filters

out in Figs. 6 and 7, which from considerations of space, must be left largely to speak for themselves. Evidently if R and f_c are known, the filter components can be readily calculated for

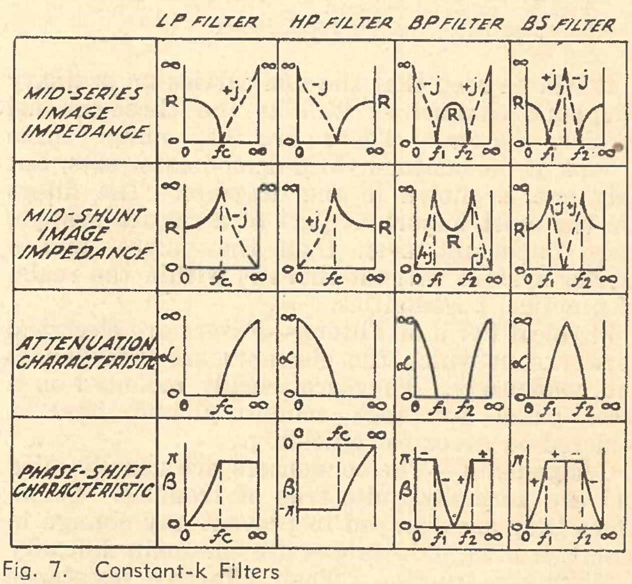


Fig. 7. Constant-k Filters

low-pass and high-pass types. The symbol f_m used in the case of band-pass and band-stop types is the geometric mean of the two cut-off frequencies.

The formulæ are arranged to bring out strongly the points of similarity between the various types. The symbol x represents the frequency divided by f_c or f_m as the case may be, and n is the fractional band width of the band-pass and band-stop filters. All graphs in this paper are merely illustrative; accurate curves will be found in the text books referred to in the bibliography.

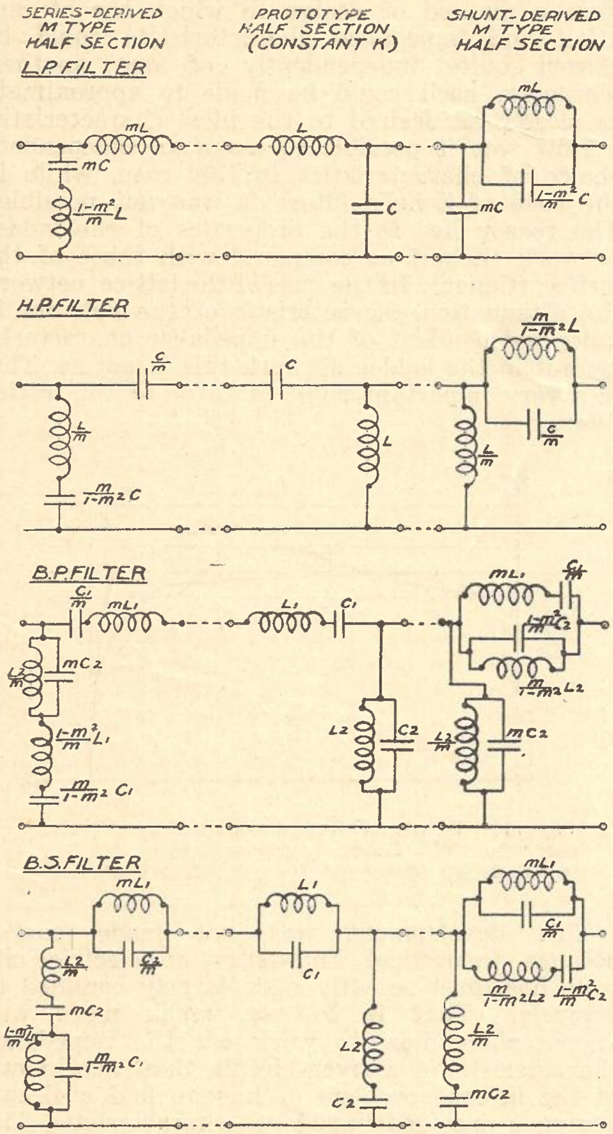


Fig. 8. M-type Filters

M-type Filters.—The m-type filter is an improvement on the constant-k as regards sharpness of cut-off and closeness of approximation to the nominal characteristic impedance. This is illustrated in Fig. 9. Fig. 8 shows the constitution of the eight kinds of m-type. From each type of constant-k filter two kinds of m-type filters are derived, the series-derived and the shunt-derived m-type, and thus there is a total of eight to be tabulated. "Series-derived" merely means that one of the impedances of the derived half-section is the same as the mid-series impedance of the constant-k half section. The other impedance of the derived half-section is as given in Fig. 9. Similar considerations apply to the meaning and properties of shunt-derived half-sections. Fig. 8 shows very clearly by the way in which the derived filters are joined to the parent half-section which impedances are matched.

From the derived half-sections, full T or TT sections can be built up just as in the case of the constant-k half-sections. The parameter m which figures in these designs is a constant lying between 0 and 1, and an idea of its effect on the filter characteristics is obtained from Fig. 9. The idea of Fig. 8 was derived from a paper commonly known as "B.P.O. Red Book No. 147."

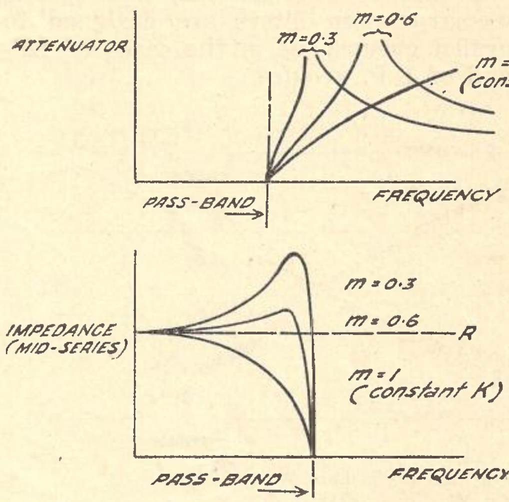


Fig. 9. Characteristics of M-type Low-pass Filters Compared with those of Constant-k Type

Types of Band-pass Filters.—There are more kinds of band-pass filters than of any other type. They may be summarised in three main families:

1. n may be varied. (Wide or narrow band).
2. For each value of n, m may be varied. (Sharp or slow cut-off, close or rough impedance matching). This takes the form of a symmetrical six-element section.
3. Two different values of m may be used for the two stop bands, giving attenuation peaks which are independently variable by variation of the two m's. This is the most general type of band-pass section, and is a dissymmetrical six-element section.

Family 3 includes the following special cases of particular importance:—(a) One attenuation peak is at zero or infinite frequency (5 elements); (b) one attenuation peak coincides with a cut-off frequency (4 elements); (c) combining (a) and (b) (3 elements).

mm′-type Filters.—Zobel has shown in 1931 that just as m-type filters are formed from constant-k filters, so the process can be repeated and mm′ filters derived from m-type filters. The characteristics approach more nearly the ideal at the expense of added complication, but it is not known that any great use has been made of this development.

Composite Filters.—The design of a filter usually includes a great deal more than the calculation of a constant-k section or an m-type section. Nearly all filters are composite, i.e., they are compounded of several sections of various types, and the art of arranging these structures is a rather complicated one. Space precludes any outline of this part of the subject, but mention should at least be made of fractional terminations of filters, the use of half-sections, and special devices necessary when filters are designed to work in parallel groups,, as in the cases of LP-HP pairs and of B.P. groups.

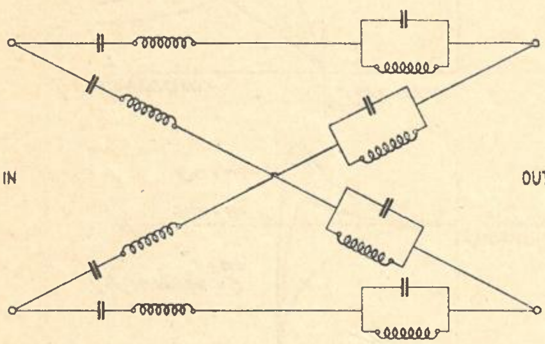


Fig. 10. Bridge or Lattice Filter, after W. Cauer; Class 3b

Recent Developments

After this inadequate summary of the Zobel type of filter, attention will now be turned to greatly improved methods of design which have recently made their appearance. First consider what is wrong with the Zobel filter. This type, of course, is the type that has been used almost exclusively in communication work up to the present, so that it is evidently of considerable merit; nevertheless, it has certain disadvantages. The chief one is that there is no possibility of an independent choice of impedance and attenuation characteristic. If the impedance characteristic of a constant-k section is not satisfactory, the designer chooses an m-type. If the impedance curve of this is satisfactory, he is obliged to accept also the m-type attenuation characteristic. There is no possibility of choosing a filter section having an impedance characteristic of the m-type and an attenuation characteristic of the constant-k type.

A most interesting development in filter design took place in 1931 with the publication of the work "Siebschaltungen" by W. Cauer, of the firm of Siemens and Halske, Berlin. This was a design handbook including 68 design charts and 14 tables, and was based on the properties of the lattice or bridge filter, of which an example is shown in Fig. 10. This work gave a method of design in which the attenuation and impedance characteristics could be chosen quite independently of one another. Moreover, each could be made to approximate as closely as desired to the ideal characteristic.

How was it possible to make an independent choice of characteristics in this case, while in the case of Zobel's filters it was not possible? The reason lies in the properties of the ladder network (Zobel) as compared with those of the lattice (Cauer). In the case of the lattice network the attenuation characteristic of the network is quite independent of the impedance characteristic but in the ladder network this is not so. This is a very important point in favor of the lattice filter.

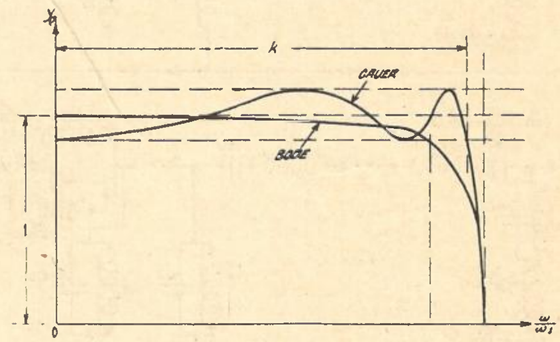


Fig. 11. Tschebyscheff Approximation used by W. Cauer, compared with H. W. Bode's Monotonic Type of Approximation

This development was not made earlier because theoretical knowledge of electric circuits has until recently been largely confined to analysis. That is to say, much more was known about how to work out the impedance characteristic of a given circuit, than was known of the inverse problem of how to find a circuit having a specified impedance characteristic. The problem of analysis was of course much easier than the inverse one of synthesis, and naturally came earlier. At the present time there are many problems of circuit synthesis still awaiting solution. There were two specific developments, however, on which Cauer's work depended, and these were Foster's Reactance Theorem, 1924, and Cauer's own paper "Vierpole," 1929. It will be advantageous to consider these briefly.

Foster's reactance theorem shows how the configuration and elements of a 2-terminal network may be worked out, provided that the impedance characteristic is expressed in the form:—

$$Z = j\omega H \frac{(\omega^2 - \omega_1^2)(\omega^2 - \omega_3^2)\ldots\ldots(\omega^2 - \omega^2_{2n-1})}{\omega^2(\omega^2 - \omega_2^2)(\omega^2 - \omega_4^2)\ldots\ldots(\omega^2 - \omega^2_{2n-2})} \qquad (1)$$

where $\omega_1, \omega_3, \omega_5$ are the frequencies at which the reactance is zero, and ω_2, ω_4, etc., those at which the reactance is infinite. H is a positive constant. If, therefore, any impedance requirements can be put into this form, the circuit having these characteristics can be readily worked out. Foster has given two forms in which the circuit can be worked out, each containing the least number of elements, and later Cauer has given two other forms, also containing the least number of elements.

Table 1
Table III. from "Siebschaltungen"
by W. Cauer

Tabelle III. Funktionen $\sqrt{\frac{Z_2}{Z_1}}$ und $\sqrt{Z_1 Z_2}$ für verschiedene Siebschaltungsklassen.
$\sqrt{\frac{Z_2}{Z_1}}$ für NDF, $\sqrt{\frac{Z_1}{Z_2}}$ für NDF*
$\sqrt{Z_1 Z_2}$ für HDF, $\frac{1}{\sqrt{Z_1 Z_2}}$ für HDF*
1) $m \frac{\lambda}{\sqrt{\lambda^2 + \omega^2_1}}$
2) $m \frac{\lambda^2 + \omega^2_a}{\lambda\sqrt{\lambda^2 + \omega^2_1}}$
3) $m \frac{\lambda(\lambda^2 + \omega^2_a)}{(\lambda^2 + \omega^2_b)\sqrt{\lambda^2 + \omega^2_1}}$
4) $m \frac{(\lambda^2 + \omega^2_c)(\lambda^2 + \omega^2_a)}{\lambda(\lambda^2 + \omega^2_b)\sqrt{\lambda^2 + \omega^2_1}}$
5) $m \frac{\lambda(\lambda^2 + \omega^2_c)(\lambda^2 + \omega^2_a)}{(\lambda^2 + \omega^2_d)(\lambda^2 + \omega^2_b)\sqrt{\lambda^2 + \omega^2_1}}$
6) $m \frac{(\lambda^2 + \omega^2_e)(\lambda^2 + \omega^2_c)(\lambda^2 + \omega^2_a)}{\lambda(\lambda^2 + \omega^2_d)(\lambda^2 + \omega^2_b)\sqrt{\lambda^2 + \omega^2_1}}$
. . . .

This theorem has been extended by Brune, Gewertz and Cauer himself to cover the case of a four-terminal network and the case where resistances appear as well as reactances. Finally in 1931, Cauer extended the theorem to cover the case of a network with n pairs of terminals.

Cauer's paper "Vierpole" had an important bearing on his subsequent method of filter design, for in it he showed that the lattice network is the most general type of symmetrical quadripole and includes all T and TT sections. He also showed that in a lattice network the attenuation and impedance characteristics could be chosen independently.

Cauer's Procedure.—The design process set out by Cauer in "Siebschaltungen" would be very lengthy to describe in detail. Briefly, his method depends on expressing the impedance and attenuation requirements in such a way that they lead to expressions for the impedances of the arms which are of the form given by Foster's reactance thearem. Then the details of the arms can be worked out. Tables 18 and 19 taken from his book show the form of the attenuation and impedance functions. They are arranged in increasing order of complexity and any impedance function required can be combined with any attenuation function. Inspection of Tables 1 and 2 will show how multiplication and division of any pair of the functions given will lead to impedance functions of the type given in (1) above.

Table 2
Table IV. from "Siebschaltungen"
by W. Cauer

Tabelle IV. Funktionen $\sqrt{\frac{Z_2}{Z_1}}$ und $\sqrt{Z_1 Z_2}$ für verschiedene Siebschaltungsklassen.
$\sqrt{Z_1 Z_2}$ für NDF, $\frac{1}{\sqrt{Z_1 Z_2}}$ für NDF*
$\sqrt{\frac{Z_2}{Z_1}}$ für HDF, $\sqrt{\frac{Z_1}{Z_2}}$ für HDF*
α) $\mu \frac{1}{\sqrt{\lambda^2 + \omega^2_1}}$
β) $\mu \frac{(\lambda^2 + \omega^2_\alpha)}{\sqrt{\lambda^2 + \omega^2_1}}$
γ) $\mu \frac{(\lambda^2 + \omega^2_\alpha)}{\sqrt{\lambda^2 + \omega^2_1}\,(\lambda^2 + \omega^2_\beta)}$
δ) $\mu \frac{(\lambda^2 + \omega^2_\alpha)(\lambda^2 + \omega^2_\gamma)}{\sqrt{\lambda^2 + \omega^2_1}\,(\lambda^2 + \omega^2_\beta)}$
ε) $\mu \frac{(\lambda^2 + \omega^2_\alpha)(\lambda^2 + \omega^2_\gamma)}{\sqrt{\lambda^2 + \omega^2_1}\,(\lambda^2 + \omega^2_\beta)(\lambda^2 + \omega^2_\delta)}$
ζ) $\mu \frac{(\lambda^2 + \omega^2_\alpha)(\lambda^2 + \omega^2_\gamma)(\lambda^2 + \omega^2_\varepsilon)}{\sqrt{\lambda^2 + \omega^2_1}\,(\lambda^2 + \omega^2_\beta)(\lambda^2 + \omega^2_\delta)}$
. . . .

The major part of Cauer's work is occupied with the problem of finding from the performance specification (coverage and tolerance of attenuation and impedance) which of the functions in Tables 1 and 2 (and their equivalents) to use. To do this he makes use of Tschebyscheff approximation parameters relating tolerance and coverage to class. By a series of frequency transformations, he has contrived that all types of filter are described in terms of the low pass filter, and so a large number of graphs provide read-made solutions to an enormous number of filter problems, which consequently do not need to be worked out from fundamentals.

The filters given by Cauer's procedure have serious practical disadvantages. It is their lattice form which makes it possible to choose attenuation and impedance characteristics independently of one another, but it is this same lattice form which provides a very difficult problem when it comes to making up the network. A lattice filter is really a bridge which is balanced to a greater or less extent at the various frequencies, and where, as in the Cauer filter, this form is used to give all the attenuation required in one section, extremely high precision in the filter elements is required as well as the usual precautions of a.c. bridge measuring technique such as shielding. In fact, some of the Cauer sections are quite unrealisable in the form of lattice sections.

Having worked out a filter according to the Cauer procedure, it is therefore necessary to convert the final lattice network into an equivalent network which can be more readily realised. Apparently it is always possible to find an equivalent ladder circuit for any lattice, and this appears to be the best form to use. The attenuation being divided up between a number of ladder sections in series, makes it possible then to construct it without either close tolerances for the elements, or exacting precautions as to shielding. The equivalent circuits used by Cauer are not of the ladder type, but employ transformers to derive unusual-looking circuits which appear to have nothing to recommend them beyond patent considerations.

Bode's Work.—Fig. 11 shows the kind of characteristic used by Cauer. The way in which the curve approximates to the ideal is known as a Tschebyscheff approximation. It is a very good form of approximation except where phase distortion is important, in which case it is necessary to keep the attenuation characteristic as smooth as possible. This is an aspect of the matter taken up by H. W. Bode of the Bell Telephone Laboratories, whose work on the subject was published last year. His method of procedure is to specify the impedance and phase characteristic, and the type of approximation which this leads to is shown in Fig. 11. It will be seen that whereas Bode's procedure results in better phase distortion characteristics than Cauer's, his attenuation characteristic does not approximate to the ideal over so wide a frequency range.

Future Developments.—From what has been said, it should be clear that in spite of the great achievements of the Zobel filter, it is highly desirable to improve on it, and that important improvements in design are at present being made which have as their basis some fairly involved theoretical developments.

Situated at the end of such a fruitful decade as the last has been in network theory and filter design, it is impossible not to speculate on possible future developments. Evidently the most important results can be expected from extensions to the theoretical side of our knowledge of networks, though how this will be easy with such things as matrices and Jacobian elliptic functions already figuring in the theory is hard to see. Improvements in materials and components should make many results attainable which are at present out of reach, and piezoelectric quartz elements and magnetostrictive rods will possibly find more extended application in the systems of the future.

Perhaps the most interesting developments will be seen in the conjunction of the thermionic vacuum tube with filter circuits. The thermionic tube in its dynatron connection at present offers negative resistance for use as a circuit element, and through its agency negative inductance and negative capacity are at least theoretically available. Considering the results obtained by the use of positive inductance and capacity up to the present, there seems every reason to expect that a very big field will lie waiting to be explored if ever negative inductance and negative capacity can be made available for use as circuit elements.

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