COURSE OF TECHNICAL INSTRUCTION

BASIC MATHEMATICS

FOR LINEMEN-IN-TRAINING

Engineering Training Section, Headquarters, Postmaster-General's Department, Melbourne C.2.

BASIC MATHEMATICS

FOR LINEMEN-IN-TRAINING

ISSUED 1953

Contents:—

TABLES.

UNITS OF LENGTH.

12 inches = 1 foot
3 feet = 1 yard
22 yards = 1 chain
10 chains = 1 furlong
8 furlongs = 1 mile
7·92 inches = 1 link
100 links= 1 chain
80 chains= 1 mile

UNITS OF AREA.

144 square inches = 1 square foot
9 square feet = 1 square yard
484 square yards = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile

UNITS OF VOLUME

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

In addition, the following units of volume are used, especially when dealing with liquids -

2 pints = 1 quart
4 quarts = 1 gallon

UNITS OF WEIGHT.

16 ounces = 1 pound
14 pounds = 1 stone
2 stones = 1 quarter
28 pounds = 1 quarter
56 pounds = 1/2 hundred-weight
4 quarters = 1 hundred-weight
112 pounds = 1 hundred-weight
20 hundred-weights = 1 ton

UNITS OF TIME.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year

ABBREVIATIONS.

The following abbreviations are in general use -

Units	Abbreviation	Units	Abbreviation
Inch	in. or "	Stone	st.
Foot and feet	ft. or '	Hundredweight	cwt.
Yard	yd.	Second	sec.
Square inch	sq. in.	Minute	min.
Square foot	sq. ft.	Hour	hr.
Square yard	sq. yd.	Week	wk.
Cubic inch	cub. in.	Month	mth.
Cubic foot	cub. ft.	Year	yr.
Cubic yard	cub. yd.	Degrees	° Used for Angles.
Pint	pt.	Minutes	'
Quart	qt.	Seconds	"
Gallon	gal.	Degrees Fahrenheit	°F Used for Temperature.
Ounce	oz.	Degrees Centigrade	°C
Pound	lb.		

GENERAL INFORMATION.

LENGTH.

Diameter of one half-penny = 1".

Twenty-two paces stepped by an average man = 1 chain (approx.)

Spacing between telephone poles (trunk route) = 44 yds. = 2 chains (approx.)

Spacing between telephone poles (sub. route) = 55 yds. = 2-1/2 chains (approx.)

AREA.

100 sq. ft. = 1 square of flooring.

VOLUME.

A common brick, with the mortar used for laying it, is about 9" long, 4-1/2" broad, and 3" deep.

1 pint of liquid = 34-1/2 cub. ins. (approx.)

1 gallon of liquid = 277-1/4 cub. ins. (approx.)

1 cub.ft. of liquid = 6-1/4 gallons (approx.)

A bag of cement contains about 1 cub. ft.

WEIGHT.

Three new pennies or five new half-pennies weigh 1 oz.

The weight of a common brick is 7 lbs.

1 pint of water weighs 20 ozs.

1 gallon of water weighs 10 lbs.

1 cub. ft. of water weighs 62-1/2 lbs. (approx.)

A bag of cement weighs about 96 lbs.

24 bags of cement weigh about 1 ton.

1 cub. yd. of dry sand weighs about 1 ton.

COMMONWEALTH OF AUSTRALIA.

Engineering Branch,
Postmaster-General's Department,
Treasury Gardens,
Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

BASIC OPERATIONS OF ARITHMETIC.

CONTENTS:

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1. INTRODUCTION.

1.1 In our everyday life we find a practical need for numbers and simple measures. Mathematics is the science of numbers and an elementary understanding of this subject is very important for a lineman. This course is not intended as a full study of the subject of mathematics but provides sufficient information to enable a lineman to make calculations of the type met with in simple lines practice.

1.2 For the purpose of this course, mathematics is divided into -

(i) Arithmetic.

(ii) Algebra.

(iii) Geometry and Mensuration.

1.3 Before commencing the course, the student should have an elementary knowledge of the principles of addition, subtraction, multiplication and division. The student should also be familiar with the more common symbols used in arithmetic. Symbols are "shorthand" methods of labelling simple processes such as -

+ for addition,

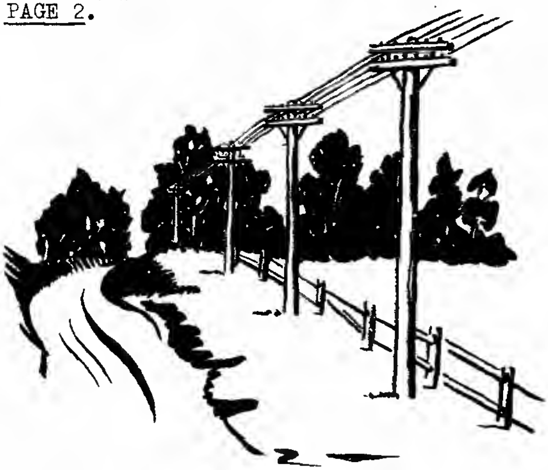
− for subtraction,

× for multiplication,

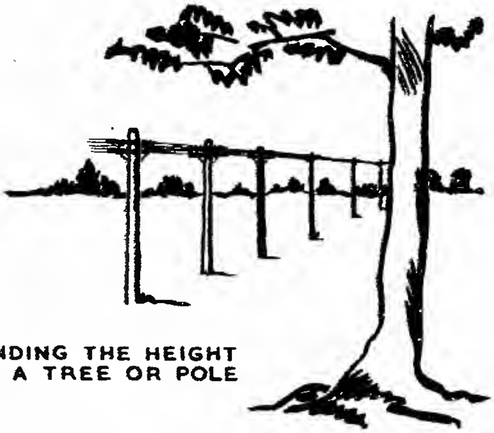
÷ for division,

= for equals sign.

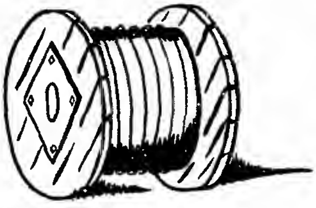
Other symbols and terms used in mathematics are explained in this course.



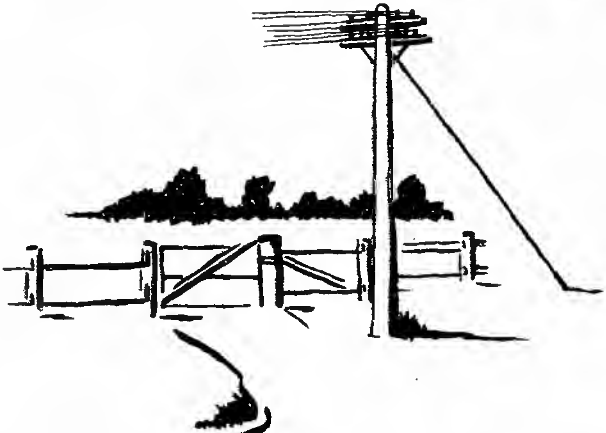
FINDING QUANTITIES OF LINE MATERIAL FOR A JOB



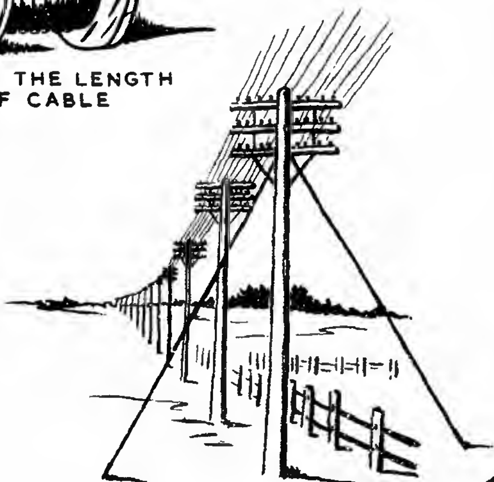
FINDING THE HEIGHT OF A TREE OR POLE



FINDING THE LENGTH OF CABLE



FINDING THE SIZE OF STAY WIRE AND STAY ROD REQUIRED



FINDING LINE OF STAY AT RIGHT ANGLE TO A ROUTE

FINDING THE VOLUME OF SOIL EXCAVATED

FINDING DISTANCE ACROSS A RIVER

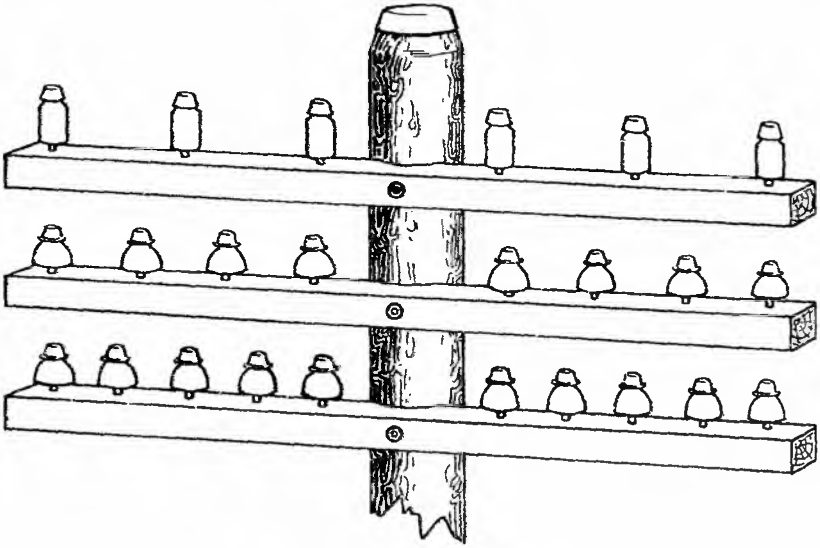
SOME TYPICAL PROBLEMS WHICH REQUIRE A KNOWLEDGE OF MATHEMATICS.

2. WHAT IS ARITHMETIC?

2.1 **Arithmetic is the most elementary form of mathematics and is the basis of all mathematics. There are four fundamental processes** - addition, subtraction, **multiplication and division - and these are applied** to the calculation of length, **area, volume, weight, time,** money, **etc.**

If we understand exactly what we are doing as we perform the basic operations of **arithmetic, we will easily understand the principles of** algebra, geometry and **mensuration.**

2.2 All arithmetic is counting, but there are many ways to count.



ILLUSTRATING METHODS OF COUNTING.

FIG. 1.

The basic counting method is to use numbers 1, 2, 3, 4, 5, 6, etc. Referring to Fig. 1 and using this method, we find there are 24 insulators on the arms.

We could count the insulators on each arm separately and add them together, for example - 6 + 8 + 10 = 24.

This is the addition method of counting.

We could count the insulators in pairs, or groups of two, and multiply the number of groups by the number in each group, for example -

12 x 2 = 24.

This is the multiplication method of counting. The number resulting from the multiplication of two or more numbers is termed the product.

Therefore, whether we count, add or multiply, it is only a different way of doing the same thing.

2.3 Just as we can count backward, we can also add backward or multiply backward.

Referring to Fig. 1, we know there are 24 insulators in all, and there are 10 on the bottom arm. This leaves 14 on the top and middle arms, for example -

24 - 10 = 14.

Subtraction is the opposite of addition.

Again, we know there are 24 insulators in all, and there are two in each group. Therefore, there are 12 groups of insulators, for example -

24 ÷ 2 = 12.

Division is the opposite of multiplication.

2.4 Therefore, counting, adding and multiplying are the only basic processes in arithmetic. Counting backwards, subtraction and division are merely their opposites.

/ 3.

3. ADDITION AND SUBTRACTION.

3.1 Before we can work with an understanding of numbers, we must realise that the numbers themselves are counters, sums and products.

We have only ten counters, namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Other numbers are made up of their sums and products, for example -

We group two tens to make twenty, three tens to make thirty, and so on.

$$57 = \overset{\text{Product}}{\overbrace{5 \times 10}} \qquad \overset{\text{Sum}}{\overbrace{+}} \qquad \overset{\text{Counter}}{\overbrace{7}} = \text{Fifty-seven.}$$

Similarly, we group ten tens to make one hundred, ten hundreds to make a thousand, and so on.

$$\begin{array}{rcccccccc} 384 & = & 3 \times 10 \times 10 & + & 8 \times 10 & + & 4 & & \\ & = & 300 & + & 80 & + & 4 & & \\ & = & \multicolumn{7}{l}{\text{Three hundred and eighty-four.}} \\ 506 & = & 500 & + & 6 & & & & \\ & = & \multicolumn{7}{l}{\text{Five hundred and six.}} \\ 1123 & = & 1000 & + & 100 & + & 20 & + & 3 \\ & = & \multicolumn{7}{l}{\text{One thousand, one hundred and twenty-three.}} \end{array}$$

If we keep this in mind, we will more easily understand what we are doing in simple arithmetic.

3.2 Simple Addition. When we add or subtract numbers, we use units, tens, hundreds, thousands, etc; for example, consider this simple problem in addition -

Problem. 237 insulators are used on one job and 562 on another. What is the total number of insulators used?

$$\left.\begin{array}{r} 237 \\ +\ \underline{562} \\ =\ 799 \end{array}\right\} \text{is the same as} \left\{\begin{array}{ccccccc} \text{Hundreds} & & \text{Tens} & & \text{Units} & & \\ 200 & & 30 & & 7 & & \\ +\ \underline{500} & & +\ \underline{60} & & +\ \underline{2} & & \\ 700 & + & 90 & + & 9 & = & 799. \end{array}\right.$$

Answer = 799 insulators.

When adding numbers, it is often necessary to "carry-over" certain quantities, for example -

Problem. Add together 237 insulators and 784 insulators.

$$\underbrace{\begin{array}{ccccccccc} \text{Thousands} & & \text{Hundreds} & & \text{Tens} & & \text{Units} & & \\ & & 200 & & 30 & & 7 & & \\ & & +\ 700 & & +\ 80 & & +\ 4 & & \\ \underline{1000} & \longleftarrow & \underline{100} & \longleftarrow & \underline{10} & \longleftarrow & \underline{\quad} & & \text{(carry over)} \\ 1000 & + & 000 & + & 20 & + & 1 & & \end{array}}_{=\ 1021.}$$

7 + 4 = 11 which is the same as 10 + 1. The 1 is entered in the units column and the 10 is carried over to the tens column.

30 + 80 + 10 = 120 which is the same as 100 + 20. The 20 is entered in the tens column and the 100 is carried over to the hundreds column.

200 + 700 + 100 = 1000 which is carried over to the thousands column.

In practice, most of this reasoning is done in the head and addition problems are usually set out thus -

$$\begin{array}{r} 237 \\ \underline{784} \\ 1021. \end{array}$$

Answer = 1021 insulators.

3.3 Simple Subtraction. In the same way, let us see the meaning of borrowing in subtraction.

Problem. 537 insulators are held in stock and 265 are withdrawn for use on a job. How many insulators are left in stock?

$$\left.\begin{array}{r} 537 \\ -\ \underline{265} \end{array}\right\} \text{is the same as} \left\{\begin{array}{r} 500 \\ -\ \underline{200} \end{array}\right. \qquad \begin{array}{r} 30 \\ -\ \underline{60} \end{array} \qquad \begin{array}{r} 7 \\ -\ \underline{5} \end{array}$$

7 - 5 = 2, but 60 cannot be taken from 30. We therefore borrow 10 tens or 100 from the 500, and then subtract, thus -

$$\begin{array}{r} 400 \\ -\ \underline{200} \\ 200 \end{array} \;+\; \begin{array}{r} 130 \\ -\ \underline{60} \\ 70 \end{array} \;+\; \begin{array}{r} 7 \\ -\ \underline{5} \\ 2 \end{array} \;=\; 272.$$

In practice, most of this subtraction is done in the head and the problem is usually set out thus -

$$\begin{array}{r} 537 \\ \underline{265} \\ 272 \end{array}$$

Answer = 272 insulators.

3.4 Remember - When adding or subtracting, we must keep the units, tens, hundreds, etc., in line. Only similar quantities can be added or subtracted.

3.5 Checking Methods. We should always check the result of our addition and subtraction, no matter how simple the problem.

(i) If we have made a mistake the first time, we might make the same mistake again if we add in the same way to check the answer. So it is better to add the column first in one direction and then check by adding in the opposite direction.

$$\begin{array}{r} \text{ADD} \\ \\ 7 \\ 9 \\ 4 \\ 6 \\ \underline{3} \\ 29 \end{array} \qquad\qquad \begin{array}{r} \text{CHECK.} \\ \underline{29} \\ 7 \\ 9 \\ 4 \\ 6 \\ 3 \end{array}$$

(ii) Another method of checking is to add the units, tens, hundreds, etc., columns separately, remembering to check each addition as previously shown.

$$\begin{array}{r} \text{ADD} \\ 247 \\ 182 \\ 349 \\ \underline{136} \\ 914 \end{array} \qquad\qquad \begin{array}{rl} \text{CHECK.} & \\ 247 & \\ 182 & \\ 349 & \\ \underline{136} & \\ 24 & \text{(adding units.)} \\ 190 & \text{(adding tens.)} \\ \underline{700} & \text{(adding hundreds.)} \\ \underline{914} & \end{array}$$

(iii) A rough check helps to prevent big mistakes. This is done by taking a quick glance at the first column, and guessing about how big the answer will be before starting to add.

$$\begin{array}{rlr} 3872 & \text{is about} & 4000 \\ 2106 & \text{is about} & 2000 \\ 1194 & \text{is about} & 1000 \\ \underline{2985} & \text{is about} & \underline{3000} \\ \text{Answer} & \text{is about} & 10000 \end{array}$$

(iv) Subtraction is checked by adding from the bottom up. The two lower numbers must add up to equal the top number.

$$\begin{array}{r} \text{SUBTRACT} \\ 427 \\ \underline{198} \\ 229 \end{array} \qquad\qquad \begin{array}{r} \text{CHECK.} \\ 427 \leftarrow \\ \left.\begin{array}{r} \underline{198} \\ 229. \end{array}\right\} \end{array}$$

/ 4.

4. MULTIPLICATION AND DIVISION.

4.1 Multiplication. Multiplication is the process by which a number is added to itself any number of times proposed. For example, $4 + 4 + 4 = 12$, therefore three fours added together equal 12.

In multiplication symbols, this is written -

$$4 \times 3 = 12,$$

because there are three fours.

The number to be multiplied is called the multiplicand.
The number by which the multiplicand is multiplied is called the multiplier.
The number resulting from the multiplication is called the product.

Multiplicand × Multiplier = Product.

Problem. Consider this simple problem in multiplication -

An aerial line route has 379 poles. On each pole there are 5 arms. How many arms are used?

This problem could be solved by the addition method of counting, for example -

(i) We could write the figure 5 down 379 times and then add all the figures together, or
(ii) We could write 379 down 5 times, and then add them together.

However, problems of this type, in which a given number is added to itself a number of times, are more easily solved by the multiplication method of counting, for example -

$$\text{Number of arms} = 379 \times 5.$$

$$\left.\begin{array}{r}379\\ \underline{\times 5}\end{array}\right\}\text{ is the same as}\left\{\begin{array}{r}300\\ \underline{\times 5}\\ 1500\end{array}\right. + \begin{array}{r}70\\ \underline{\times 5}\\ 350\end{array} + \begin{array}{r}9\\ \underline{\times 5}\\ 45\end{array} = 1895.$$

In practice, when we multiply, most of this operation is done in the head, and the problem is set out thus -

$$\begin{array}{r}379\\ \underline{5}\\ 1895\end{array}$$

Answer = 1895 arms.

Problem. When the multiplier contains more than one figure, the first number of each successive multiplication is placed directly below the number of the multiplier which we are using, as shown in this slightly harder problem -

An aerial line route has 297 poles. Each pole has 3 arms fitted and on each arm there are 12 insulators. How many insulators are used?

$$\text{Number of insulators} = 297 \times 3 \times 12$$
$$= 297 \times 36.$$

We first multiply the multiplicand by the units number of the multiplier, which is 6, in this case.

$$\begin{array}{r}297\\ \underline{36}\\ 1782\\ \underline{8910}\\ 10692\end{array}$$

Answer = 10692 insulators.

We then multiply the multiplicand by the tens number of the multiplier, which is 3, in this case.

These two products are added together to give the answer.

/ Remember

Remember - Always place the first number of each product directly below the number in the multiplier which we are using. Add the required number of noughts in each product. When multiplying by the tens number of the multiplier, add one nought; when multiplying by the hundreds number, add two noughts, and so on.

$$\begin{array}{r}347\\ \underline{407}\\ 2429\\ \underline{138800}\\ 141229\end{array}$$

When there are zeros in the multiplier, bring down the zero into the product exactly below the zero in the multiplier.

4.2 Division. Division is the process of finding how many times one number contains another. For example, 4 + 4 + 4 = 12, therefore -

12 contains 4 three times.

Expressed in symbols -

$$12 \div 4 = 3.$$

The number to be divided is called the dividend.
The number that does the dividing is called the divisor.
The number resulting from the division is called the quotient.

Division is indicated in several ways -

12 divided by 4 equals 3.	$12 \div 4 = 3$	Dividend ÷ Divisor = Quotient.
	$\begin{array}{r}3\\4\overline{)\,12}\end{array}$ or $\begin{array}{r}4)\,\underline{12}\\3\end{array}$	$\begin{array}{r}\text{Quotient}\\ \text{Divisor}\overline{)\,\text{Dividend}}\end{array}$ or $\begin{array}{r}\text{Divisor})\,\underline{\text{Dividend}}\\ \text{Quotient.}\end{array}$
One-fourth of 12 equals 3.	$\frac{12}{4} = 3$	$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient.}$
	$12/4 = 3$	Dividend /Divisor = Quotient.

Problem. Consider this simple problem in division -

A line party is required to erect 252 poles. If the party erects 6 poles per day how many days will it take to finish the job?

This problem could be solved by the subtraction method of counting.

However, problems of this type, in which we wish to find how many times one number contains another, are more easily solved by the division method of counting, for example -

252 poles ÷ 6 poles per day = ? days.

$6\overline{)\,252}$ is the same as $6\overline{)\,200 + 50 + 2}$

$$\begin{array}{lrcrcr}
 & 30 & + & 10 & + & 2\\
6\,) & \overline{200} & \overline{+} & \overline{50} & \overline{+} & \overline{2}\\
 & \underline{180} & & & & \\
 & 20 & + & 50 & & \\
 & & = & 70 & & \\
 & & & \underline{60} & & \\
 & & & 10 & + & 2\\
 & & & & = & 12\\
 & & & & & \underline{12}
\end{array}$$

Answer = 30 + 10 + 2 = $\underline{\underline{\text{42 days}}}$.

/ In

In practice, when we divide, most of this operation is done in the head and the problem is set out thus -

$$\begin{array}{r} 42 \\ 6\overline{)\,252} \\ \underline{24} \\ 12 \\ \underline{12} \end{array}$$

Answer = 42 days.

This is an example of working by long division. When the divisor is just one figure, 9 or less, all the multiplying and subtracting can be done in the head and the problem set out in short division form. Sometimes the quotient is put under the dividend in short division.

$$\begin{array}{r} 42 \\ 6\overline{)\,252} \end{array} \qquad \text{or} \qquad \begin{array}{r} 6\underline{)\,252} \\ 42 \end{array}$$

Problem. Consider another typical problem, which is an example of long division -

A total length of 10582 yards of copper wire is available at a line depot, and this length is divided into 26 sections of equal length. What is the length of each section?

Length of each section = 10582 yards ÷ 26 sections.

$$\begin{array}{r} 407 \\ 26\overline{)\,10582} \\ \underline{104} \\ 182 \\ \underline{182} \end{array}$$

Answer = 407 yards per section.

The problem comes out even without leaving a remainder. This means that 26 can be subtracted from 10582 exactly 407 times. It also means that 26 × 407 = 10582.

Remember - Division is a quick method of subtracting. In our first step of this division, we subtracted 26, four hundred times in one operation. This left us with 182. We then subtracted 26 seven more times in one operation.

We often find that division problems do not come out even. We still have a remainder after we have used the last number of the dividend. This remainder is expressed in the answer as a fraction. Fractions are discussed in Paper No. 2 of this book.

4.3 Average. We are familiar with the meaning of average through the medium of the daily newspapers where, for example, the average number of runs per innings that cricketers make, is commonly given. When a batsman makes the following scores -

69, 16, 35, 72, 13, 46, 87, 54,

then he has made a total of 392 runs and his average is -

Total number of runs ÷ Number of innings

= 392 ÷ 8

= 49.

The average is the result obtained by adding a group of numbers together and dividing by the number in the group.

Generally, the average of a set of values is only useful when it indicates a probable result or when it is used for comparison purposes.

Problem. The total time taken to erect 8 poles is 56 hours. What is the average time taken for each pole?

Average Time = Total time ÷ Number of poles

= 56 hours ÷ 8

= 7 hours per pole.

/ 4.4

4.4 Checking Methods. Problems in multiplication and division should always be checked for accuracy. For an easy check in multiplication, reverse the multiplicand and multiplier, for example -

MULTIPLY	CHECK.
347	407
407	347
2429	2849
138800	16280
141229	122100
	141229

Multiplication can also be checked by division.

For example, 347 × 407 = 141229, therefore when checking,

141229 ÷ 347 should equal 407, or
141229 ÷ 407 should equal 347.

In our check, therefore, Product ÷ Multiplicand = Multiplier, or
Product ÷ Multiplier = Multiplicand.

Similarly, problems in division can be checked by multiplication -

Dividend ÷ Divisor = Quotient,
therefore, when checking, Divisor x Quotient = Dividend.

5. BASIC OPERATIONS APPLIED TO DISTANCE, TIME, WEIGHT, ETC.

5.1 Addition and subtraction. In all addition and subtraction, the quantities added or subtracted must be the same. For example -

1 yard + 3 yards = 4 yards,
1 foot + 3 feet = 4 feet,
but 1 yard + 3 feet = ?

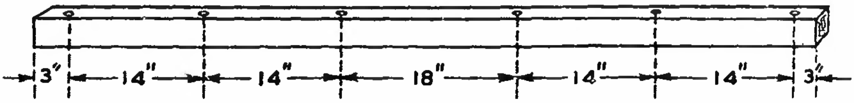
Before we can add yards and feet, we must either convert the yards to feet or the feet to yards.

1 yard = 3 feet.
Therefore, 1 yard + 3 feet = 3 feet + 3 feet = 6 feet,
or 1 yard + 3 feet = 1 yard + 1 yard = 2 yards.

Similarly, inches and feet cannot be added to or subtracted from one another, although both are measures of distance. Before we can add inches and feet, we must convert the inches to feet or the feet to inches.

Similar reasoning applies when adding and subtracting all units of distance, time, weight, money, etc. This is shown in the following problems.

Problem No. 1. What is the length of the arm shown in Fig. 2.



WHAT IS LENGTH OF ARM?
FIG. 2.

The symbol " means inch or inches.
The symbol ' means foot or feet.

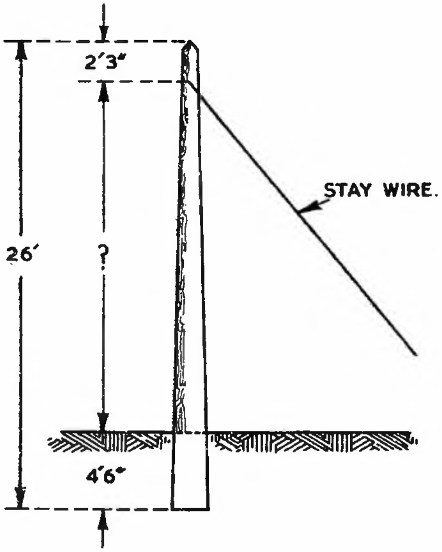
As there are 12" in 1', 14" = 1'2"
18" = 1'6".

Length of arm =	Inches		Feet		Inches
	3	or	0		3
	14		1		2
	14		1		2
	18		1		6
	14		1		2
	14		1		2
	3		0		3
	80"		5'	+	20"

= 5' + 12" + 8"
= 5' + 1' + 8"
= 6' + 8"
= 6'8".

Answer = 80" or 6'8".

Problem No. 2. A pole 26' in length is set in the ground to a depth of 4'6". A stay wire is attached 2'3" from the top of the pole. What is the distance from the ground to the point of attachment of the stay wire? (See Fig. 3.)



WHAT IS HEIGHT TO ATTACHMENT OF STAY WIRE?

FIG. 3.

Unknown distance = 26' - 2'3" - 4'6".

$$\left.\begin{array}{r} 26'0" \\ -\ 2'3" \end{array}\right\} \text{is the same as} \left\{\begin{array}{rr} & 25'12" \\ - & 2'\ 3" \\ = & 23'\ 9" \end{array}\right.$$

23'9"
4'6"
19'3"

Answer = 19'3".

Problem No. 3. Three men work the following periods, respectively, to complete a job -

12 hours 55 minutes; 10 hours 25 minutes; 9 hours 50 minutes.

If the estimated time for the job is 31 hrs. 30 mins., by what period does the total time taken for the job exceed the estimated time?

12 hrs. 55 mins.
10 hrs. 25 mins.
9 hrs. 50 mins.

Total time = 31 hrs. + 130 mins.

= 31 hrs. + 2 hrs. 10 mins.

= 33 hrs. + 10 mins.

Total time exceeds estimated time by 33 hrs. 10 mins. - 31 hrs. 30 mins.

$$\left.\begin{array}{r} \text{33 hrs. 10 mins.} \\ \text{31 hrs. 30 mins.} \end{array}\right\} \text{is the same as} \left\{\begin{array}{r} \text{32 hrs. 70 mins.} \\ \text{31 hrs. 30 mins.} \\ \hline \text{1 hr. 40 mins.} \end{array}\right.$$

Answer = 1 hr. 40 mins.

5.2 Multiplication.

Problem No. 4. Five lengths each 28'8" long are cut from a coil of wire. What is the total length of wire cut off?

Total length = 28'8" x 5

$$\begin{array}{rcr} 28' & & 8" \\ \underline{5} & & \underline{5} \\ 140' & + & 40" \end{array}$$

= 140' + 3'4"

Answer = **143'4".**

Problem No. 5. A workman works 7 hrs. 21 mins. per day for a five-day week. How many hours does he work per fortnight?

1 fortnight = 5 x 2

= 10 working days.

.˙. Total working hours = 7 hrs. 21 mins. x 10.

= 70 hrs. + 210 mins.

= 70 hrs. + 3 hrs. 30 mins.

Answer = **73 hrs. 30 mins.**

5.3 Division.

Problem No. 6. A length of wire 13'8" long is cut into 4 equal lengths. What is the length of each section?

13'8" ÷ 4

$$\begin{array}{rr} & 3'5" \\ 4) & 13'8" \\ & \underline{12'} \\ & 1'8" \\ = & 20" \\ & \underline{20"} \end{array}$$

4 divides into 13'8", 3 times, leaving 1'8" remainder.

Before we can divide 4 into 1'8" we must convert to inches. 1'8" = 12 + 8 = 20".

4 then divides into 20", 5 times.

Answer = **3'5".**

Problem No. 7. Four men each work a period of 7 hrs. 30 mins. to erect five poles. What is the average time taken for each pole?

Total time = 7 hrs. 30 mins. x 4

= 30 hrs.

Average Time = Total Time ÷ Number of poles.

= 30 hrs. ÷ 5

= **6 hrs. per pole.**

/ Problem

Problem No. 8. Seven men spend a total of 60 hrs. 5 mins. to complete a job. If each man works an equal period of time, for how long does each work?

60 hrs. 5 mins. $\div$ 7

8 hrs. 35 mins.
7) 60 hrs. 5 mins.
56 hrs.
4 hrs. 5 mins.
= 240 mins. + 5 mins.
= 245 mins.
21
35
35

Answer = 8 hrs. 35 mins.

Problem No. 9. The estimated time for a job is 108 hrs. 30 mins. How many men each working for a day period of 7 hrs. 45 mins. must be put on the job to complete it in one working day?

108 hrs. 30 mins. $\div$ 7 hrs. 45 mins.

A problem of this nature is more easily solved by the use of fractions or decimals which are covered in Paper No. 2.

6. TEST QUESTIONS.

1. An aerial line route is divided into five sections. Each section requires 176, 288, 192, 204 and 160 insulators respectively. How many insulators are used on the job?
2. 873 insulators are held in stock. The quantities required for three jobs are 238, 374 and 268, respectively. Can these quantities be supplied from stock?
3. 3 cwt. of copper wire are held in stock, and 178 lbs. are required for a certain job. What quantity is left in stock?
4. Four coils of 200 lb. H.D.C. wire of equal length weigh a total of 472 pounds. What is the weight of each coil?
5. What whole numbers will divide into 360 without leaving a remainder?
6. How many inches in a mile?
7. How many yards in 9684 inches?
8. An aerial line route is two miles long. How many poles at intervals of 55 yards are required?
9. Eight aerial line pairs are to be erected over a distance of 7 miles. The spacing between poles is 44 yards and each pole has 2 arms. Calculate the quantities of poles, arms, insulators and length of copper wire required for the job.

END OF PAPER.

COMMONWEALTH OF AUSTRALIA.

Engineering Branch,
Postmaster-General's Department,
Treasury Gardens,
Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

PAPER NO. 2.
PAGE 1.

FRACTIONS, DECIMALS AND PERCENTAGE.

CONTENTS:

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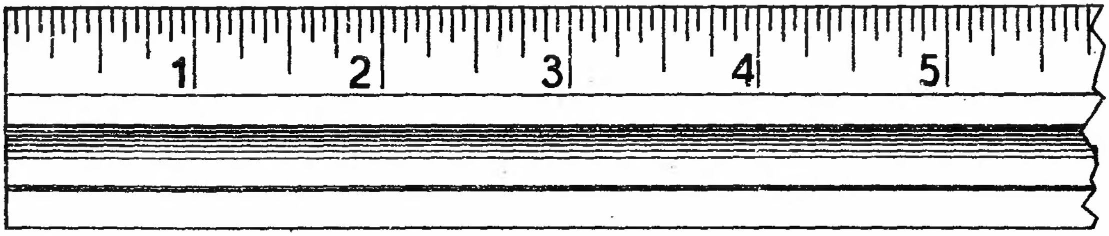
1. FRACTIONS.

1.1 To enable us to get exact information about things, we have units of measurement, for example - foot, pound, second. However, the things we measure often do not come out an exact number of units. There is usually a portion of the unit to take into consideration. This portion is known as a fraction. It is an accurate measure for something that is smaller than one complete unit.

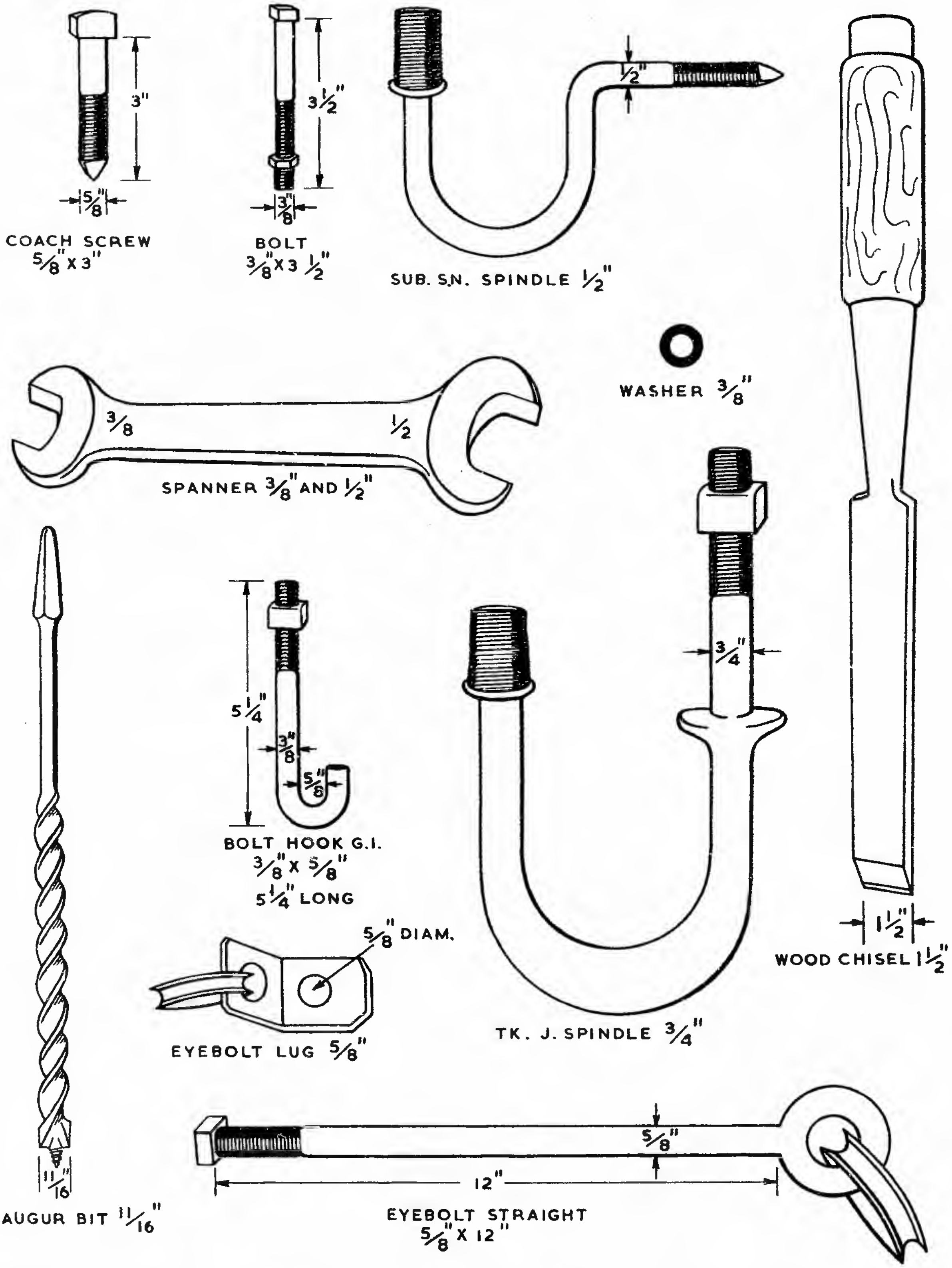
To measure fractions, we divide the unit of measurement into a number of parts. Then we see how many of these parts the portion of the unit takes.

To help in the understanding of fractions, look at a rule marked in inches. We find that each inch is, in turn, further subdivided into equal divisions. There may be 2, 4, 8, 10, 16, etc., subdivisions in each inch. This rule measures distance in fractions of an inch.



RULE IN INCHES.

FIG. 1.



USE OF FRACTIONS TO DESCRIBE TOOLS AND EQUIPMENT USED IN AERIAL LINE CONSTRUCTION.
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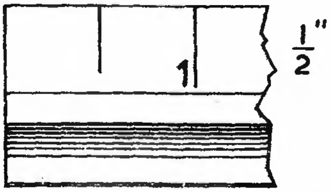
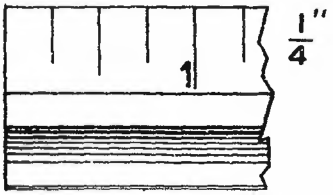
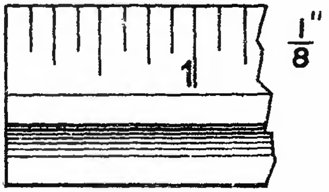


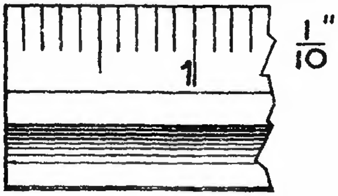
When an inch is divided into 2 equal parts, each part is called half an inch.



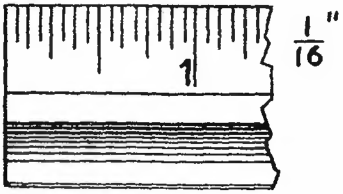
For 4 equal parts, each part is one fourth (or one quarter) of an inch.



For 8 equal parts, each part is one eighth of an inch.



For 10 equal parts, each part is one tenth of an inch.



For 16 equal parts, each part is one sixteenth of an inch.

FRACTIONS OF AN INCH.

FIG. 2.

Suppose we use a rule marked in fourths of an inch to measure, say, the width of a chisel blade, (see Fig. 3), and find that the blade covers three of the small divisions. We could now say that the blade is "three out of four parts" of an inch wide. For convenience, we call it three-fourths or three-quarters of an inch. In number form we could show it as "3 4ths". This last way of writing is not very clear. When the 3 and 4 are written too close it looks like 34. To keep the 3 and 4 apart, a sloping line is put in like this, 3/4ths. Usually the "ths" is omitted and the fraction is shown as -

3/4 inch or 3/4".

Sometimes the 3 is put over the 4, thus $\frac{3''}{4}$.

If the distance measured were one inch and one-quarter of an inch, we show it in the number form as -

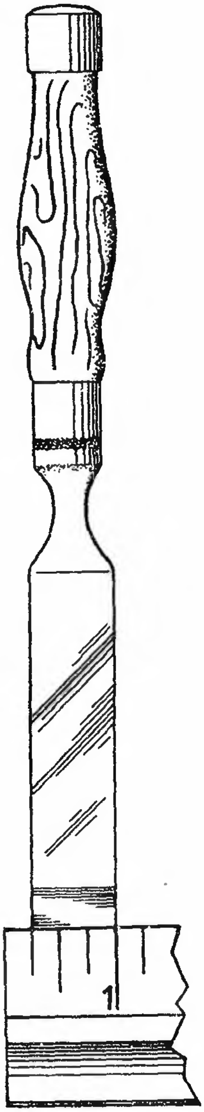
1-1/4" or $1\,\frac{1''}{4}$.

It is important to note that the line between 1 and 1/4 in 1-1/4 is put simply to separate the whole number from the fraction and must not be confused with a minus sign.

When writing a fraction, we have to ask ourselves two questions -

(i) How many of the parts are we using? This gives us the first (or top) number of the fraction, and is called the numerator.

(ii) How many parts make up the unit? This gives us the second (or bottom) number of the fraction, and is called the denominator.



MEASURING FRACTIONS.

FIG. 3.

/1.2

1.2 Proper and Improper Fractions. A proper fraction is one in which the numerator is less than the denominator, such as 7/8. Its value is always less than 1.

An improper fraction is one in which the numerator is greater than the denominator such as 29/8. Its value is greater than 1 and so, strictly speaking, it is not a fraction at all, hence the term improper. It actually comprises a whole number and a proper fraction.

Frequently, the things we measure are greater than one unit of measurement. They measure "so many units and a little bit more". Take for example, a measurement of 3-5/8 inches.

A fractional reading of this type is known as a mixed fraction. This fraction is read "three and five-eighths". The word "and" means addition. So this fraction reads "three plus five-eighths", and is the same as 3 inches + 5/8 inch.

When using a mixed fraction in a problem, we often convert it to an improper fraction. To convert a mixed fraction to an improper fraction, multiply the whole number by the denominator and add it to the numerator.

For example, convert 3-5/8 to an improper fraction -

$$\begin{aligned} 3\text{-}5/8 &= 3 + \frac{5}{8} \\ &= \frac{3 \times 8}{8} + \frac{5}{8} \\ &= \frac{24}{8} + \frac{5}{8} \\ &= \frac{29}{8}. \end{aligned}$$

$$\text{Similarly, } 7\text{-}1/4 = \frac{29}{4},$$

$$\text{and, } 5\text{-}1/10 = \frac{51}{10}.$$

Improper fractions are not easy to visualise or to measure with. When the result of a problem is an improper fraction, it is usually better to change it to a mixed fraction. This is done by dividing the numerator by the denominator.

For example, convert $\frac{103}{8}$ to a mixed fraction -

$$\begin{array}{r} 12 \\ 8\overline{)103} \\ \underline{8} \\ 23 \\ \underline{16} \\ 7. \end{array}$$

This is an example of a division problem that does not come out even. After we have used the last number of the dividend, there is still a remainder of 7. This remainder is the numerator of the fraction, and the divisor is the denominator. The fraction is then added to the quotient.

$$\text{Therefore, } \frac{103}{8} = 12\frac{7}{8} \text{ or } 12\text{-}7/8.$$

$$\text{Similarly, } \frac{65}{4} = 16\,\frac{1}{4} \text{ or } 16\text{-}1/4,$$

$$\text{and, } \frac{17}{10} = 1\frac{7}{10} \text{ or } 1\text{-}7/10.$$

/1.3

1.3 Reduction of Fractions. When the numerator and denominator of a fraction are each multiplied by the same number, the value of the fraction is not altered.

$$\text{Thus} \quad \frac{1}{2} = \frac{2}{4} \text{ or } \frac{3}{6} \text{ or } \frac{4}{8}\text{, and so on,}$$

$$\text{and} \quad \frac{3}{8} = \frac{6}{16} \text{ or } \frac{9}{24} \text{ or } \frac{12}{32}\text{, and so on.}$$

Similarly, when the numerator and denominator of a fraction are each divided by the same number, the value of the fraction is not altered.

$$\text{Thus} \quad \frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4},$$

$$\text{and} \quad \frac{90}{100} = \frac{90 \div 10}{100 \div 10} = \frac{9}{10}.$$

A fraction such as $\frac{875}{1000}$ is clumsy and should be expressed as simply as possible, thus -

$$\frac{875 \div 5}{1000 \div 5} = \frac{175}{200}$$

$$\frac{175 \div 5}{200 \div 5} = \frac{35}{40}$$

$$\frac{35 \div 5}{40 \div 5} = \frac{7}{8}.$$

7/8 is not capable of further similar treatment, and thus 7/8 represents $\frac{875}{1000}$ in its simplest form, or, as it is called - "reduced to its lowest terms".

Remember - All answers to questions involving fractions must always be reduced to their lowest terms.

To reduce a fraction to its lowest terms, divide the numerator and denominator by the highest number that will exactly divide into each of them. This is known as the Highest Common Factor abbreviated to H.C.F.

For small numbers the H.C.F. is found by inspection, thus -

the H.C.F. of 8 and 16 is 8,

the H.C.F. of 12 and 30 is 6,

the H.C.F. of 8, 16, and 20 is 4.

When the H.C.F. is not obvious it is found as described in paragraph 3.2 of Paper No. 3.

2. ADDITION AND SUBTRACTION OF FRACTIONS.

2.1 Adding and subtracting fractions is similar to adding and subtracting simple whole numbers.

In all addition and subtraction, the units must be the same. Unlike quantities cannot be added or subtracted. In the case of fractions, this means that only those fractions which have the same denominator can be added or subtracted.

2.2 Addition. To add fractions which have the same denominator, it is simply necessary to add the numerators, and place the result over the common denominator. The answer must be reduced to its lowest terms. For example -

$$\frac{1}{16} + \frac{1}{16} = \frac{1+1}{16} = \frac{2}{16} = \frac{1}{8}.$$

$$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}.$$

$$\frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3}.$$

$$\frac{7}{8} + \frac{3}{8} = \frac{7+3}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}.$$

To add fractions which have different denominators, it is necessary to convert them so that they have a common denominator. Then add the numerators of these converted fractions and place the result over the common denominator. For example -

Problem. What is the sum of $\frac{1''}{4}$ and $\frac{3''}{8}$?

$$\frac{1}{4} = 1 \text{ of size } \frac{1}{4}$$

$$\frac{3}{8} = 3 \text{ of size } \frac{1}{8}.$$

To add these, we must either convert fourths to eighths or eighths to fourths -

$$\frac{1}{4} = \frac{2}{8} = 2 \text{ of size } \frac{1}{8}$$

$$\frac{3}{8} = 3 \text{ of size } \frac{1}{8}.$$

$$\text{Therefore, } \frac{1}{4} + \frac{3}{8} = 5 \text{ of size } \frac{1}{8},$$

$$\text{Answer} = \frac{5''}{8}.$$

We could convert the fractions to any size we like, but to simplify calculations, we select a size so that each fraction is a whole number of that size. It is usual, in practice, to select for the common denominator the lowest number into which each of the individual denominators will divide exactly. This is known as the Lowest Common Denominator abbreviated to L.C.D.

For small numbers the L.C.D. is found by inspection; thus for denominators -

2 and 3, the L.C.D. is 6,

2 and 8, the L.C.D. is 8,

2, 4, 8 and 16, the L.C.D. is 16.

When the L.C.D. is not obvious, it is found in the same manner as described for the Lowest Common Multiple in paragraph 3.3 of Paper No. 3.

Problem. An aerial line route is pegged out in four sections which measure 1/2, 3/4, 5/8 and 7/16 mile respectively. What is the total length of the route?

$$\text{Length in miles} = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16}.$$

We first convert into fractions with the same denominator. When we multiply all the denominators together, we get a common denominator -

$$2 \times 4 \times 8 \times 16 = 1024.$$

But this common denominator is higher than necessary and this causes extra work. We therefore find the L.C.D. which, by inspection, is 16 in this case.

We then convert each fraction to this common denominator, and add -

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16}$$

$$= \frac{8}{16} + \frac{12}{16} + \frac{10}{16} + \frac{7}{16}$$

$$= \frac{37}{16}$$

$$\text{Answer} = \underline{\underline{2\frac{5}{16}\text{ miles.}}}$$

2.3 Subtraction. To subtract fractions which have the same denominator, simply subtract the numerators and place the result over the common denominator. The answer must be reduced to its lowest terms. For example -

$$\frac{3}{8} - \frac{1}{8} = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}.$$

$$\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}.$$

$$\frac{7}{8} - \frac{3}{8} - \frac{1}{8} = \frac{3}{8}.$$

To subtract fractions which have different denominators, first convert them so that they have a common denominator. Then subtract the numerators of these converted fractions and place the result over the common denominator.
For example -

$$\frac{1}{2} + \frac{3}{4} - \frac{5}{8} - \frac{7}{16}$$

$$= \frac{8}{16} + \frac{12}{16} - \frac{10}{16} - \frac{7}{16}$$

$$= \frac{20-17}{16}$$

$$= \frac{3}{16}.$$

2.4 Mixed Fractions. Adding and subtracting mixed fractions is similar to adding and subtracting proper fractions. Mixed fractions need not be changed to improper fractions before adding or subtracting. For example -

Problem. An aerial line route consists of two sections which are 3-3/4 and 6-7/8 miles respectively. A man on pole inspection covers a distance of 7-1/2 miles. What length of route remains to be inspected?

$$\text{Length in miles} = 3\frac{3}{4} + 6\frac{7}{8} - 7\frac{1}{2}.$$

Remember, there are inferred plus or minus signs between the whole numbers and the fractions.

$$= 3 + \frac{3}{4} + 6 + \frac{7}{8} - 7 - \frac{1}{2}.$$

In such a problem it makes no difference in what order the additions or subtractions are made.

$$= 3 + 6 - 7 + \frac{3}{4} + \frac{7}{8} - \frac{1}{2}.$$

Add or subtract the whole numbers first and then the fractions, remembering to find the L.C.D.

$$= 2 + \frac{6}{8} + \frac{7}{8} - \frac{4}{8}$$

$$= 2 + \frac{9}{8}$$

$$= 2 + 1\frac{1}{8}$$

$$\text{Answer} = \underline{\underline{3\frac{1}{8}\text{ miles.}}}$$

When subtracting mixed fractions, it is often necessary to subtract a large fraction from a smaller one. For example -

Problem. An aerial line is 4-1/4 miles long. A fault exists 7/8 mile from the testing station. How far is the fault from the distant end?

$$4\frac{1}{4} - \frac{7}{8}$$

$$= 4 + \frac{2}{8} - \frac{7}{8}$$

$$= 4 - \frac{5}{8}.$$

Then we borrow 1 from the whole number as in normal subtraction -

$$= 3 + 1 - \frac{5}{8}$$

$$= 3 + \frac{8}{8} - \frac{5}{8}$$

$$\text{Answer} = \underline{\underline{3\frac{3}{8}\text{ miles.}}}$$

3. MULTIPLICATION AND DIVISION OF FRACTIONS.

3.1 Multiplication. Whenever we see a fraction in a problem we know the numerator is to be divided by the denominator, for example -

$$\frac{5}{8} \text{ means } 5 \div 8.$$

When we multiply two fractions, we are multiplying two problems in division -

$$\frac{5}{8} \times \frac{3}{4} \text{ means } 5 \div 8 \times 3 \div 4.$$

The quickest way to perform a series of multiplication and division is -

(i) multiply the multipliers,

(ii) multiply the divisors,

(iii) divide the product of the multipliers by the product of the divisors.

Therefore, in the multiplication of fractions, the quickest way to perform the multiplication is to multiply the numerators to find the numerator of the product, and multiply the denominators to find the denominator of the product -

$$\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}.$$

3.2 Division. Division is the opposite of multiplication. When we multiply by $\frac{3}{4}$, we multiply by 3 and divide by 4. To do the opposite, or divide by $\frac{3}{4}$, we divide by 3 and multiply by 4.

$$\frac{5}{8} \div \frac{3}{4} \text{ means } 5 \div 8 \div 3 \times 4.$$

Therefore, to divide a fraction by a fraction, turn the second fraction upside down and multiply.

$$\frac{5}{8} \div \frac{3}{4} \text{ is the same as } \frac{5}{8} \times \frac{4}{3}$$

$$= \frac{20}{24}$$

$$= \frac{5}{6}.$$

3.3 Whole numbers can be multiplied or divided in the same way. A whole number can be written as a numerator with a denominator of 1.

(i) $60 \times \frac{3}{4}$

$$= \frac{60}{1} \times \frac{3}{4}$$

$$= \frac{180}{4}$$

$$= 45.$$

(ii) $60 \div \frac{3}{4}$

$$= \frac{60}{1} \times \frac{4}{3}$$

$$= \frac{240}{3}$$

$$= 80.$$

/Similarly

Similarly, a fraction can be multiplied or divided by a whole number.

(iii) $\frac{3}{4} \times 60$

$= \frac{3}{4} \times \frac{60}{1}$

$= \frac{180}{4}$

$= 45.$

(iv) $\frac{3}{4} \div 60$

$= \frac{3}{4} \times \frac{1}{60}$

$= \frac{3}{240}$

$= \frac{1}{80}.$

3.4 Mixed fractions must be changed to improper fractions before they can be used in multiplication or division.

(i) $16\frac{1}{2} \times 2\frac{3}{4}$

$= \frac{33}{2} \times \frac{11}{4}$

$= \frac{363}{8}$

$= 45\frac{3}{8}.$

(ii) $16\frac{1}{2} \div 2\frac{3}{4}$

$= \frac{33}{2} \div \frac{11}{4}$

$= \frac{33}{2} \times \frac{4}{11}$

$= \frac{132}{22}$

$= 6.$

3.5 Multiplication and division of fractions, therefore, are just a series of ordinary multiplications and divisions. When we have many fractions to multiply and divide, we invert the divisors, and then multiply all numerators and all denominators -

$$\frac{1}{2} \times \frac{5}{8} \div \frac{5}{16} \times \frac{1}{2} \div \frac{3}{4} = ?$$

Invert the divisors.

$$\frac{1}{2} \times \frac{5}{8} \times \overset{\downarrow}{\frac{16}{5}} \times \frac{1}{2} \times \overset{\downarrow}{\frac{4}{3}} = ?$$

3.6 Cancellation. Fractions that are the result of multiplication may have some rather large numbers, for example -

$$\frac{1}{2} \times \frac{5}{8} \times \frac{16}{5} \times \frac{1}{2} \times \frac{4}{3} = \frac{320}{480}.$$

This may be reduced to its lowest terms (see paragraph 1.3). When the numerator and denominator have factors in common, these are cancelled out.

$\frac{320}{480}$ therefore simplifies to $\frac{2}{3}$.

However, there is no need to wait for a complicated result before cancelling. A multiplication problem is usually cancelled before the multiplication is done. This saves much unnecessary multiplication and reduction of the result.

$$\frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{16}}}{\underset{1}{\cancel{5}}} \times \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{3}.$$

/The

The product of the denominators of the first two fractions = 16 which cancels with the numerator of the third fraction.

The numerator of the second fraction cancels with the denominator of the third fraction.

The denominator of the fourth fraction divides into the numerator of the fifth fraction twice.

The problem now simplifies to $\frac{2}{3}$.

Consider, now, this problem which was introduced in paragraph 5.3 of Paper No. 1 -

Problem. The estimated time for a job is 108 hrs. 30 mins. How many men, each working for a day period of 7 hrs. 45 mins. must be put on the job to complete it in one working day?

$$\text{Number of men} = 108 \text{ hrs. } 30 \text{ mins. } \div 7 \text{ hrs. } 45 \text{ mins.}$$

$$= 108\tfrac{1}{2} \div 7\tfrac{3}{4}$$

$$= \frac{217}{2} \div \frac{31}{4}$$

$$= \frac{217}{2} \times \frac{4}{31}$$

$$= \frac{\overset{7}{\cancel{217}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{31}}}$$

$$= 7 \times 2$$

$$\text{Answer} = \underline{\underline{14 \text{ men.}}}$$

4. DECIMALS.

4.1 Often we can choose the total number of parts into which a unit is divided. This is true of all measuring devices. When the total number of parts, that is, the denominator, is 10, 100, 1000, etc. we can write the fraction in a simpler form. A fraction which has for its denominator, 10, 100, 1000, etc., is called a decimal fraction, or simply a decimal. All other fractions are, by way of distinction, called vulgar fractions.

The decimal system of measuring is similar to that used in counting with ordinary numbers.

In counting, we group things together always by tens. Ten units make ten, ten tens make a hundred, and so on. In this way we can measure things much larger than the unit of measurement.

But decimals are designed for the opposite purpose, that is, for measuring things smaller than a unit. To measure such things, we do not need to group units together. We do the opposite, that is, take a unit and break it up into smaller pieces. This is done by the same rule as for counting - always by tens. First of all we take a unit and break it up into ten equal parts. Then each of these parts is further subdivided into ten equal parts. We could go on like this for as long as we like but usually two or, at the most, three decimal places are sufficient for most practical purposes.

/Fig. 4.

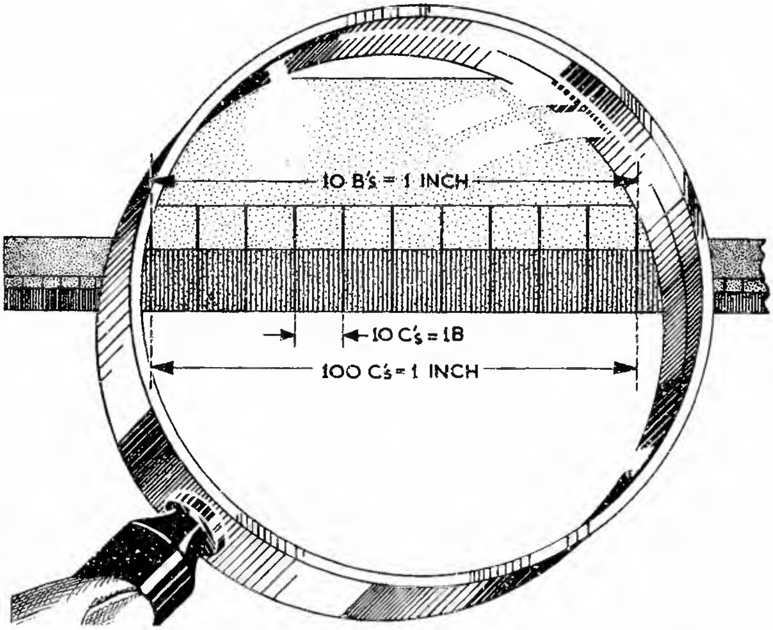


FIG. 4. DECIMALS OF AN INCH.

Fig. 4 shows a magnification of one inch. An inch is divided into ten equal parts, B. Each part B is divided into ten equal parts, C. One inch contains 100 C's. Each B is, therefore, 1/10 part of an inch or .1 inch, and each C is 1/100 part of an inch or .01 inch.

4.2 To save the trouble of writing the denominators of decimal fractions, a method of notation is used, which is explained by the following examples -

$\frac{1}{10}$ is denoted by .1, the dot being called the "decimal point" or, more simply, the "point". The number .1 is read as "point one".

$\frac{2}{10}$ is denoted by .2, and read as "point two".

$\frac{1}{100}$ is denoted by .01, usually read as "point nought one".

$\frac{1}{1000}$ is denoted by .001, usually read as "point nought nought one".

One place decimals are called tenths.
Two place decimals are called hundredths.
Three place decimals are called thousandths.

When there is a number in the numerator for every nought in the denominator, we simply put a decimal point before the numerator, for example -

$$\frac{75}{100} = .75$$

$$\frac{625}{1000} = .625$$

When this is not the case, we insert a nought for every missing number in the numerator and put a decimal point over the 1, for example -

$$\frac{3}{100} = \frac{03}{100} = .03$$

$$\frac{25}{1000} = \frac{025}{1000} = .025$$

/Table 1.

$\frac{1}{10} = .1$	$\frac{1}{100} = .01$	$\frac{1}{1000} = .001$
$\frac{2}{10} = .2$	$\frac{2}{100} = .02$	$\frac{2}{1000} = .002$
$\frac{3}{10} = .3$	$\frac{3}{100} = .03$	$\frac{3}{1000} = .003$
$\frac{4}{10} = .4$	$\frac{4}{100} = .04$	$\frac{4}{1000} = .004$
$\frac{5}{10} = .5$	$\frac{5}{100} = .05$	$\frac{5}{1000} = .005$
$\frac{6}{10} = .6$	$\frac{6}{100} = .06$	$\frac{6}{1000} = .006$
$\frac{7}{10} = .7$	$\frac{7}{100} = .07$	$\frac{7}{1000} = .007$
$\frac{8}{10} = .8$	$\frac{8}{100} = .08$	$\frac{8}{1000} = .008$
$\frac{9}{10} = .9$	$\frac{9}{100} = .09$	$\frac{9}{1000} = .009$

TABLE 1. DECIMAL TABLE.

4.3 Noughts added after a decimal do not alter its value, for example -

.7, .70, .700 are all equal.

$$.7 = \frac{7}{10}$$

$$.70 = \frac{70}{100} = \frac{7}{10}$$

$$.700 = \frac{700}{1000} = \frac{7}{10}.$$

A whole number can be written in decimal form by placing one or more noughts after the decimal point. For example, 4 can be written 4.0, 4.00, or 4.000, etc.

A number such as 4-35/100 is written in decimal form as 4.35, and this is read as "four point three five".

A decimal is often written with a nought in front of the decimal point. This nought draws attention to the decimal point, and prevents it from being overlooked, for example -

.7 is written 0.7.

5. CONVERSION OF FRACTIONS AND DECIMALS.

5.1 Both the fraction and decimal systems are used because each has some advantages -

(i) Simple fractions like 1/2, 1/4, 3/4, etc., are easy to visualise.

(ii) Fractions are used to indicate a division -

$$5 \div 8 = \frac{5}{8}\,; \qquad 12 \div 4 = \frac{12}{4}.$$

(iii) Decimals are often easier to add, subtract, multiply and divide. They permit us to save one or more steps in our arithmetic.

Because we use both the fraction and the decimal systems, it is often necessary to convert from one to the other.

5.2 Converting Fractions to Decimals. To convert a vulgar fraction to a decimal expression, reduce the fraction to its lowest terms, and then divide the denominator into the numerator. The decimal points go right after the numerator in the division. The quotient gives the decimal expression required, for example -

Problem. Convert 9" to a decimal fraction of 1'.

$$9'' = \frac{9}{12} \text{ of 1 foot.}$$

$$\frac{9}{12} = \frac{3}{4} \qquad \begin{array}{r} 4\overline{)3.00} \\ 0.75 \end{array}$$

Answer = 0.75'.

Some decimal fractions never come out even, no matter how far we carry out the division. However, we can still use this method to get as accurate an answer as we need, for example -

Problem. Convert 5" to a decimal fraction of 1'.

$$5'' = \frac{5}{12} \text{ of 1 foot}$$

$$\begin{array}{r} 0.4166\cdot\text{etc.} \\ 12\overline{)5.00000000} \\ \underline{4.8} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8. \end{array}$$

No matter how far we carry out this division, there will always be a remainder. However, in practice, there is no need for an answer that is more accurate than we can actually use. Therefore, we usually limit the answer to two or three decimal places.

When the third figure is 5 or more, we add 1 to the figure before it, for example -

0.416 is taken as 0.42,
but 0.414 would be taken as 0.41.

When we require greater accuracy, the answer may be extended to three decimal places. Then 0.4166 is taken as 0.417.

Answer = 0.4', 0.42' or 0.417' depending on the degree of accuracy required.

/5.3

5.3 Converting Decimals to Fractions. Decimals are changed to fractions by using the decimal number as the numerator. A one is placed in the denominator directly below the decimal point. A nought is placed in the denominator below each figure or nought in the decimal.

For example – .03 becomes $\frac{.03}{100}$.

Then the decimal point and any noughts following immediately to the right of it are removed from the fraction.

This gives us $\frac{3}{100}$.

Similarly .875 = $\frac{875}{1000}$, which reduced to its lowest terms, equals $\frac{7}{8}$ (see Paragraph 1.3).

5.4 Practical Use of Decimals. It is often necessary to express a given quantity as a fraction (or decimal) of another given quantity of the same kind. Some typical examples are shown.

(i) In the case of 1'3",

$3" = \frac{3}{12} = \frac{1}{4}$ of 1'.

Therefore, 1'3" is written as $1\frac{1}{4}$ ft.,

but $\frac{1}{4} = 0.25$,

therefore, 1'3" is the same as $\underline{\underline{1.25'}}$.

(ii) Express 5 days 6 hours as a decimal quantity of a week.

As there are 24 hours in a day,

5 days 6 hours = $5\frac{6}{24}$ days

= 5.25 days,

and, as there are 7 days in a week,

5 days 6 hours = $\frac{5.25}{7}$ week.

= $\underline{\underline{0.75 \text{ week.}}}$

(iii) When a quantity is given in decimal form, its value is obtained as shown in the following example –

Find the number of feet and inches in 0.75 yard.

0.75 yard = 0.75 × 3 feet = 2.25 feet.

0.25 feet = 0.25 × 12 inches = 3.0 inches.

Therefore, 0.75 yard = $\underline{\underline{\text{2 feet 3 inches.}}}$

6. BASIC OPERATIONS IN ARITHMETIC APPLIED TO DECIMALS.

6.1 Addition. When adding, decimals are set down one under another, with the decimal points in a vertical line. The figures are then added up in the same manner as whole numbers.

$$\begin{array}{rr} & 0.4 \\ & \underline{0.2} \\ = & 0.6 \end{array} \qquad \begin{array}{rr} & 0.15 \\ & \underline{0.65} \\ = & 0.80 \\ = & 0.8 \end{array} \qquad \begin{array}{rr} & 0.042 \\ & \underline{0.975} \\ = & 1.017 \end{array}$$

When the decimals to be added differ in the number of figures after the decimal point, they are arranged as before with their decimal points in a vertical line. The addition is commenced from the right, care being taken to add only those figures that are in the same vertical column.

$$\begin{array}{rl} & 0.5 \\ & \underline{0.875} \\ = & 1.375 \end{array} \qquad \begin{array}{rl} & 7.1 \\ & 0.55 \\ & \underline{0.875} \\ = & 8.525 \end{array}$$

Problem. A workman's hours of duty for four working days are as follows -

8 hrs.,
8 hrs. 48 mins.,
7 hrs. 21 mins., and
7 hrs. 45 mins.

What are the total number of hours worked? Give the answer in decimal form.

$$\begin{array}{lcl} 8 \text{ hrs.} & = & 8.0 \text{ hrs.} \\ 8 \text{ hrs. } 48 \text{ mins.} = 8\frac{48}{60} \text{ hrs.} & = & 8.8 \text{ hrs.} \\ 7 \text{ hrs. } 21 \text{ mins.} = 7\frac{21}{60} \text{ hrs.} & = & 7.35 \text{ hrs.} \\ 7 \text{ hrs. } 45 \text{ mins.} = 7\frac{45}{60} \text{ hrs.} & = & \underline{7.75} \text{ hrs.} \\ \text{Total time worked} & = & 31.90 \text{ hrs.} \\ & = & \underline{\underline{31.9 \text{ hrs.}}} \end{array}$$

6.2 Subtraction. The subtraction of decimals is similar to the subtraction of whole numbers. The decimals are arranged so that the decimal points are in the same vertical line.

To subtract 0.35 from 0.47,

$$\begin{array}{rl} & 0.47 \\ & \underline{0.35} \\ = & 0.12 \end{array}$$

To find the difference between decimals which have different numbers of figures after the decimal points, the method is to make the number of figures after the points equal in both cases, by adding noughts to the decimal with the smaller number of figures.

To subtract 0.375 from 0.5,

$$\begin{array}{rl} & 0.500 \\ & \underline{0.375} \\ = & 0.125 \end{array}$$

To subtract 0.5 from 0.75,

$$\begin{array}{rl} & 0.75 \\ & \underline{0.50} \\ = & 0.25 \end{array}$$

/To

To subtract a decimal from a whole number, the whole number is written with a decimal point to the right, and as many noughts added after this decimal point as there are figures in the decimal to be subtracted.

To subtract 0.375 from 2,

$$\begin{array}{r} 2.000 \\ \underline{0.375} \\ = 1.625 \end{array}$$

To subtract 3.05 from 4,

$$\begin{array}{r} 4.00 \\ \underline{3.05} \\ = 0.95 \end{array}$$

Problem. The estimated time for a job is 83.5 man-hours. If the actual time taken is 88.375 man-hours, by how much does the actual time exceed the estimated time?

$$\begin{array}{r} 88.375 \\ \underline{83.500} \\ = \quad 4.875 \end{array}$$

Answer = 4.875 man-hours.

6.3 **Multiplication.** When multiplying decimals, we could first convert them into fractional form, thus -

$$0.4 \times 0.6 = \frac{4}{10} \times \frac{6}{10} = \frac{24}{100} = 0.24$$

$$0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = 0.06$$

$$1.25 \times 0.3 = \frac{125}{100} \times \frac{3}{10} = \frac{375}{1000} = 0.375$$

Omitting the intermediate steps, -

$$0.4 \times 0.6 = 0.24$$

$$0.2 \times 0.3 = 0.06$$

$$1.25 \times 0.3 = 0.375$$

These figures show that the number of figures or places after the decimal point in the product equals the total number of decimal figures in the multiplicand and multiplier.

Hence, in multiplication involving decimals, we first find the product as if there were no decimals, and then fix the decimal point.

Problem. 200 lb. H.D.C. wire has an approximate resistance of 4.4 ohms per single wire mile. What is the resistance of 13-1/4 miles of wire?

$$\text{Total resistance} = 13\frac{1}{4} \times 4.4$$

$$= 13.25 \times 4.4$$

$$\begin{array}{r} 13.25 \\ \underline{4.4} \\ 5300 \\ \underline{5300} \\ 58.300 \end{array} \text{ ohms.}$$

Answer = 58.3 ohms.

In this problem, there are two decimal figures in the multiplicand, and one in the multiplier, that is, a total of three. We therefore place the decimal point in the product so that there are three figures on the right of the decimal point.

6.4 Division.

(i) The process of dividing a decimal expression by a whole number is the same as for ordinary division, but, in dividing decimals, we have to decide the position of the decimal point in the quotient. This decimal point must be put directly in line with the point in the dividend.

To divide 34.5 by 3,

$$\begin{array}{r} 3\underline{)34.5} \\ 11.5 \end{array}$$

Answer = $\underline{\underline{11.5}}$

(ii) In cases where the division does not work out evenly and we wish to carry out the division further, add as many noughts as required to the figures which come after the decimal point in the dividend.

To divide 34.5 by 4,

$$\begin{array}{r} 4\underline{)34.500} \\ 8.625 \end{array}$$

Answer = $\underline{\underline{8.625}}$

(iii) Some combinations of numbers will never come out even, no matter how far we carry out the division. But we can use this method to get as accurate an answer as we need.

To divide 34.5 by 7,

$$\begin{array}{r} 7\underline{)34.5000000} \\ 4.9285714 \text{ etc.} \end{array}$$

There is no need for a quotient that is more accurate than we can actually use.

Therefore, Answer = $\underline{4.929,\ 4.93 \text{ or } 4.9}$, depending on the degree of accuracy required.

(iv) When the divisor ends in nought, the noughts may be removed, and the decimal points moved accordingly in the dividend.

$7575 \div 30$ is the same as $757.5 \div 3$.

$7575 \div 2500$ is the same as $75.75 \div 25$.

(v) When the divisor has a decimal, it is first converted to a whole number. This is done by shifting the decimal point to the end of the number. The decimal point in the dividend must also be shifted an equivalent number of places. We then proceed as for ordinary division.

(a) $5.72 \div 5.2$

$= 5.2\overline{)5.72}$

$= 52.\overline{)57.2}$

$$\begin{array}{r} 1.1 \\ 52\overline{)57.2} \\ \underline{52} \\ 52 \\ \underline{52} \end{array}$$

Answer = $\underline{\underline{1.1}}$

(b) $3 \div 0.05$

$= 0.05\underline{)3}$

$= 5.\underline{)300}$

$$\begin{array}{r} 5\underline{)300} \\ 60 \end{array}$$

Answer = $\underline{\underline{60}}$.

7. PERCENTAGE.

7.1 There is another system besides fractions and decimals to indicate a portion of a total. It is called percentage. Percentage is a fraction with 100 parts in the denominator. For example -

$\frac{5}{100}$ means 5 parts in every 100 parts or 5 per cent.

"Per cent" is short for the Latin "per centum" which means "out of one hundred".

This is usually expressed in symbol form as 5%.

5% of a total therefore simply means five hundredths of the total.

We mainly use percentage when we want to compare the size of things. It is often easier to visualise 32% of a total than 8/25 of the total.

Often the percentage will require more than two figures to be accurate, for example, 32-1/2% or 32.5%.

When we compare a large object with a smaller one, we get a percentage greater than 100%. When the larger object is 1-1/2 times the size of a smaller one, it is equivalent to 150%.

Percentage values can also be written as a decimal. Regardless of the size of the percentage, we find the decimal value by moving the decimal point two places to the left, for example -

$$5\% = \frac{5}{100} = .05$$

$$32\frac{1}{2}\% = \frac{32.5}{100.0} = .325$$

$$150\% = \frac{150}{100} = 1.5$$

7.2 All problems in percentage have the same three elements -

the %, the part, the whole.

We must always know two of them to find the third. The part and the whole are related to the % and 100 as follows -

$$\frac{\%}{100} = \frac{\text{Part}}{\text{Whole.}}$$

By cross multiplication (see paragraph 5.1 of Paper No. 3),

$$\% \times \text{Whole} = \text{Part} \times 100.$$

From this equation, we obtain the following -

$$\% = \frac{\text{Part} \times 100}{\text{Whole.}}$$ (When we divide each side by whole).

$$\text{Part} = \frac{\text{Whole} \times \%}{100}$$ (When we divide each side by 100).

$$\text{Whole} = \frac{\text{Part} \times 100}{\%}$$ (When we divide each side by %).

If we remember these relations, we can work any problem in percentage.

/Problem

Problem. We may wish to compare the size of two units.

What percentage of marks is obtained by a student who obtains 150 out of a possible 200.

$$\frac{\%}{100} = \frac{150}{200}$$

$$\% = \frac{150 \times 100}{200}$$

$$= \underline{\underline{75}}.$$

Problem. We may wish to find a certain percentage of a whole unit. What is 30% of 200?

$$\frac{30}{100} = \frac{\text{Part}}{200}$$

$$\text{Part} = \frac{200 \times 30}{100}$$

$$= \underline{\underline{60}}.$$

Problem. We may wish to find an unknown number, of which some known number is a certain percentage.

70 is 35% of what?

$$\frac{35}{100} = \frac{70}{\text{Whole}}$$

$$\text{Whole} = \frac{70 \times 100}{35}$$

$$= \underline{\underline{200}}.$$

8. TEST QUESTIONS.

1. Draw lines 2/3 foot and 3/4 foot in length, which line is the longer? By how much?

2. Which is the larger 2.3 inches or 2-1/4 inches, and by how much?

3. How many times does 1.3 inches go into 10-1/2 inches, and what is left over? Give your answer in decimals and also in fractions.

4. Add together the following distances and express the answer as a decimal fraction of a mile –
   3 chains, 600 yards, 100 feet, 48 inches.

5. Express the following as percentages of 5000 –

   (i) 200, (ii) 350, (iii) 750, (iv) 4000, (v) 4875.

6. Express $\frac{11}{55}$ and $\frac{55}{11}$ as percentages.

7. 100 lb. H.D.C. wire has an approximate resistance of 8.8 ohms per single wire mile. What is the resistance of 200 yards of wire?

END OF PAPER.

COMMONWEALTH OF AUSTRALIA.

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Postmaster-General's Department,
Treasury Gardens,
Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

FURTHER OPERATIONS IN ARITHMETIC.

CONTENTS:

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1. PLANNING A PROBLEM.

1.1 The various problems attempted up to date have shown the basic operations of arithmetic, and these operations have been applied to problems involving whole numbers and fractions.

In practice, it is often necessary to apply a number of different operations in the solution of a problem. The problem then involves the addition, subtraction, multiplication and division of whole numbers and fractions in different combinations.

In a more complex problem of this nature we must plan the problem carefully before attempting the working out. This will -

(i) ensure that we do everything that must be done,

(ii) enable us to keep track of the units so that the answer will have true meaning,

(iii) save many unnecessary operations.

The sequence in the solution of a problem is -

(i) to read the problem carefully and understand the nature of the problem,

(ii) to arrange the information in a logical order so that it points towards the solution,

(iii) to solve the calculation by the simplest means.

1.2 This paper deals with the solution of more advanced problems in arithmetic. It introduces a number of time-saving short cuts in the solution of the problems and explains some further operations in arithmetic.

2. BRACKETS.

2.1 Brackets are used to indicate the order of operations in a mathematical problem involving a number of different operations. Brackets used in mathematics are of various forms, the most common shapes being -

$$(\quad), \qquad \{ \quad \}, \qquad [\quad].$$

2.2 Necessity for Brackets. Consider, for example, this problem which requires the use of addition, subtraction and multiplication for its solution.

Problem. Twenty poles are erected on an aerial line route. Five of these poles have 3 arms each and the remainder 2 arms each. How many arms are used?

5 poles have 3 arms each,

therefore 5 poles have $5 \times 3 = 15$ arms.

The remainder $= 20 - 5 = 15$, have 2 arms each,

therefore 15 poles have $15 \times 2 = 30$ arms.

Total = 15 arms + 30 arms

= 45 arms.

In arithmetical language, this problem could be written thus -

Number of arms = $5 \times 3 + 20 - 5 \times 2$.

In a series problem of this nature we must be careful of all signs of operation which differ from the sign ahead, to ensure that the operations are done in the correct order.

For example, unless we refer back to the original problem, it is not apparent whether we have to add 3 and 20 or subtract 5 from 20, before we multiply. To eliminate this danger, brackets are used to enclose the smaller problems within the larger one.

The above problem is then written -

Number of arms = $5 \times 3 + (20 - 5) \times 2$.

Although it is usual to perform multiplication and division before addition and subtraction, we can avoid any possibility of confusion by also enclosing the multiplication problems in brackets -

$$\begin{aligned}\text{Number of arms} &= [5 \times 3] + [(20 - 5) \times 2] \\ &= [5 \times 3] + [15 \times 2] \\ &= 15 + 30 \\ &= \underline{\underline{45 \text{ arms}}}.\end{aligned}$$

/ 2.3

2.3 Use of Brackets. When we set out the problem in arithmetic symbols, we can apply many time-saving short cuts.

In a series of additions and subtractions, we can -

(i) add all the numbers to be added,

(ii) add all the numbers to be subtracted,

(iii) from the total of the additions subtract the total of the subtractions.

For example -

$$16 + 8 - 4 + 2 - 1 = ?$$

is the same as $(16 + 8 + 2) - (4 + 1)$

$$= 26 - 5$$

$$= 21.$$

The effect of placing a negative sign before brackets is shown in the following example -

The expression $26 - 4 - 1 = 21$ signifies in mathematical language that when from 26 we subtract 4, and then from the remainder, subtract 1, we have 21 as the answer.

The expression $26 - (4 + 1) = 21$ signifies that when from 26 we subtract the sum of 4 and 1, we are left with 21 as the answer.

Evidently, the expressions -

$$26 - 4 - 1 \text{ and } 26 - (4 + 1)$$

are equal.

This shows that whenever a number or fraction is put in a bracket which is preceded by a negative sign, the sign of the number or fraction must be altered. Similarly, when a number or fraction is removed from a bracket which is preceded by a negative sign, the sign of the number or fraction must be altered. For example -

$$21 - (4 + \frac{3}{4}) = 21 - 4 - \frac{3}{4}.$$

$$15 - (9 - 3) - (6 + 1) = 15 - 9 + 3 - 6 - 1.$$

$$15 + 3 - (9 + 6 + 1) = 15 + 3 - 9 - 6 - 1.$$

Similarly, in a series of multiplications and divisions, we can -

(i) multiply all the multipliers together,

(ii) multiply all the divisors together,

(iii) divide the product of the multipliers by the product of the divisors.

/ For

For example -

$$16 \div 8 \times 4 \div 2$$

is the same as $(16 \times 4) \div (8 \times 2)$

$$= 64 \div 16$$

$$= 4.$$

Addition and subtraction are opposite processes. So are multiplication and division. This is why they can be combined in short-cut methods.

When addition and subtraction are combined with multiplication and division, we must be more careful. In the absence of brackets, we always do the multiplications and divisions first, for example -

(i) $6 + 4 \times 2$

$= 6 + 8$

$= 14$

(Right).

$6 + 4 \times 2$

$= 10 \times 2$

$= 20$

(Wrong).

(ii) $6 + 4 \div 2$

$= 6 + 2$

$= 8$

(Right).

$6 + 4 \div 2$

$= 10 \div 2$

$= 5$

(Wrong).

Even though it is agreed to dc multiplication and division first, a problem of this type may arise -

$$36 \div 4 \times 3.$$

In such cases, the operation to be done first must be indicated by brackets.

(iii) $(36 \div 4) \times 3$

$= 9 \times 3$

$= 27.$

(iv) $36 \div (4 \times 3)$

$= 36 \div 12$

$= 3.$

When there are no brackets, we must refer to the conditions of the problem to find the order of operations.

2.4 Omission of Multiplication Sign. Frequently, in problems with brackets, the multiplication sign is omitted for simplicity between a figure and brackets or between two brackets. For example -

(i) $3\ (4 + 3)$ is the same as $3 \times (4 + 3)$

$= 3 \times 7$

$= 21.$

(ii) $(4 + 2)\ \ (7 \div 3)$ is the same as $(4 + 2) \times (7 \div 3)$

$$= \frac{6}{1} \times \frac{7}{3}$$

$$= 14.$$

/ 3.

3. FACTORISING.

3.1 After we get mechanically accurate and fast with the basic operations in arithmetic, we can start using short cuts. Most short cuts are based on factorising, which is the process of splitting a number into factors.

The factors of a number are the numbers which, when multiplied together, give the number as their product. For example, 3 × 7 = 21, therefore 3 and 7 are factors of 21.

Factorising a number is simplified when the following rules are remembered -

- (i) 2 is a factor when the number is even, that is, when the last digit is divisible by 2,
- (ii) 3 is a factor when the sum of the digits is divisible by three,
- (iii) 4 is a factor when the last two digits are divisible by four,
- (iv) 5 is a factor when the number ends in 5 or 0,
- (v) 9 is a factor when the sum of digits is divisible by 9,
- (vi) 10 is a factor when the number ends in 0.

3.2 Highest Common Factor. Examples of factorising are -

$$210 = 2 \times 3 \times 5 \times 7 \qquad \begin{array}{r} 2\underline{)\ 210} \\ 3\underline{)\ 105} \\ 5\underline{)\ \ 35} \\ 7 \end{array} \qquad \text{and} \qquad 330 = 2 \times 3 \times 5 \times 11 \qquad \begin{array}{r} 2\underline{)\ 330} \\ 3\underline{)\ 165} \\ 5\underline{)\ \ 55} \\ 11. \end{array}$$

It is noted that 2, 3 and 5 are common to both the numbers 210 and 330, and so are called Common Factors.

The product, 2 × 3 × 5 = 30, is termed the Highest Common Factor, abbreviated to H.C.F., of 210 and 330.

The H.C.F. of two or more numbers is the highest number that will exactly divide into each of them.

For small numbers, the H.C.F. may be found by inspection, but when the H.C.F. is not obvious, the following method is used -

Divide the greater of the two numbers by the lesser, and the divisor by the remainder, repeating this second step until no remainder is left. The last divisor is the H.C.F. required.

Problem. Reduce the fraction $\frac{375}{1000}$ to its lowest terms.

First find the H.C.F. -

$$\begin{array}{llll} 375) & 1000(2 & & \\ & \underline{750} & & \\ & 250) & 375(1 & \\ & & \underline{250} & \\ & & 125) & 250(2 \\ & & & \underline{250} \end{array}$$

Therefore 125 is the H.C.F. of 375 and 1000.

Then divide the numerator and denominator of the fraction by the H.C.F. -

$$\frac{375}{1000} = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

Answer = $\underline{\underline{3/8}}$.

/ 3.3

3.3 **Lowest Common Multiple.** Any one number which is exactly divisible by a second number is termed a multiple of this second number. Because 12 is exactly divisible by 3, 12 is a multiple of 3.

In cases where several numbers are multiplied together, the product is a common multiple of each of the numbers multiplied. For example -

$$2 \times 4 \times 8 \times 10 = 640.$$

Therefore, 640 is a common multiple of 2, 4, 8 and 10.

In problems involving fractions it is often necessary to find the Lowest Common Multiple, abbreviated to L.C.M., of two or more numbers.

The L.C.M. of two or more numbers is the lowest number into which these two or more numbers will divide exactly.

For small numbers, the L.C.M. may be found by inspection, but when the answer is not obvious, the following method is used -

(i) Write down the numbers side by side.
(ii) Determine the lowest number which divides exactly into one or more of the numbers. Then divide each number by this divisor if it will leave no remainder. As each number is divided, draw a line through it.
(iii) On the next line, write down side by side the quotients and the numbers that are not exactly divisible by the divisor. When the divisor does not divide evenly, bring down those numbers that were not divided.
(iv) Repeat this process until a line of ones remains.
(v) The L.C.M. is obtained by multiplying the divisors together.

Problem. Find the L.C.M. of 8, 20 and 30.

$$\begin{array}{l}
2\underline{)\ \cancel{8},\ \ \cancel{20},\ \ \cancel{30}} \\
2\underline{)\ \cancel{4},\ \ \cancel{10},\ \ 15} \\
2\underline{)\ \cancel{2},\ \ \ 5,\ \ 15} \\
3\underline{)\ 1,\ \ \ 5,\ \ \cancel{15}} \\
5\underline{)\ 1,\ \ \ \cancel{5},\ \ \ \cancel{5}} \\
\ \ \underline{)\ 1,\ \ \ 1,\ \ \ 1}
\end{array}$$

$$\text{L.C.M.} = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 120.$$

3.4 **Short Cuts in Multiplication.**

(i) Any number ending in nought has 10 as a factor. When it ends in two noughts 100 is a factor, and so on -

$$5 \times 10 = 50$$
$$2 \times 100 = 200$$
$$8 \times 1000 = 8000.$$

So when we have a multiplier ending in one or more noughts, we can multiply by the first factor. We then multiply by the second factor by simply adding the same number of noughts to our first product. For example -

$$757 \times 200 = 757 \times 2 \times 100$$
$$= 1514 \times 100$$
$$= 151400.$$

/ (ii)

(ii) When the multiplier is 10, 100, 1000, etc., we can perform the entire multiplication by simply adding the same number of noughts to the multiplicand -

$$659 \times 100 = 65900.$$

(iii) It is often easier to divide by a small number than to multiply by a large one. Many numbers have factors that make this possible -

50 = 1/2 of 100
33-1/3 = 1/3 of 100
25 = 1/4 of 100
12-1/2 = 1/8 of 100.

To multiply by any of these numbers, multiply by 100 first. Then divide by the denominator of the fraction -

```
  25 × 274
= 1/4 × 100 × 274
= 1/4 of 27400
=       4)27400
          6850.
```

3.5 Short Cuts in Division.

(i) Sometimes it is easier to factorise the division and do two or more short divisions instead of one long division. For example -

$$17451 \div 63.$$

```
Long Division                     Short Division

        277
63 ) 17451                        7 ) 17451
     126                          9 )  2493
      485                               277
      441
       441
       441
```

(ii) When the divisor and the dividend both end in one or more noughts, we can simplify the division by first dividing both divisor and dividend by 10, 100 or 1000, etc. For example -

7100 ÷ 20 is the same as 710 ÷ 2.

7100 ÷ 200 is the same as 71 ÷ 2.

7100 ÷ 2000 is the same as 71 ÷ 20.

(iii) Some numbers have 100 and a fraction as factors. (See Paragraph 3.4 (iii).) When these are used as divisors, it is sometimes easier to divide the dividend by 100 and multiply by the denominator of the fraction.

$$6700 \div 25.$$

```
        268
25 ) 6700                  6700 ÷ 100 × 4
     50
     170                 = 67 × 4
     150
      200                =   268.
      200
```

/ 4.

4. SQUARES AND SQUARE ROOTS.

4.1 In some problems it is necessary to find the square or the square root of a certain number. Typical problems which require a knowledge of squares and square roots are given in Papers Nos. 5 and 6.

4.2 Squares. When a number is multiplied by itself, the resulting product is called the square of the number.

$2 \times 2 = 4$, therefore 4 is the square of 2 or, expressed in another way -

2 squared = 4.

To indicate that a number is squared, a small figure 2 is placed just above the number on the right of it, thus -

$$2^2 = 4.$$

Similarly -

$3 \times 3 = 3^2 = 9$, therefore 9 is the square of 3.

$20 \times 20 = 20^2 = 400$, therefore 400 is the square of 20.

$99 \times 99 = 99^2 = 9801$, therefore 9801 is the square of 99.

A short cut method of squaring a number is shown in the following examples. This method is given as a matter of interest and is an alternative method to the more usual process of multiplying the number by itself.

(i) What is the square of 78?

Bring the number to the nearest multiple of 10, and note the number which must be added to do this -

$$78 + 2 = 80.$$

Then subtract from the number to be squared the number which was previously added -

$$78 - 2 = 76.$$

Multiply the two results -

$$76 \times 80 = 6080.$$

Then add to this product the square of the number which was added and subtracted -

$$6080 + 2^2$$

$$= 6080 + 4.$$

$$\text{Therefore } 78^2 = \underline{\underline{6084}}.$$

(ii) What is 41 squared?

In this case, we bring to the nearest multiple of 10 by subtracting -

$$41 - 1 = 40.$$

$$\text{Then } 41 + 1 = 42.$$

$$\text{Therefore } 41^2 = 42 \times 40 + 1^2$$

$$= 1680 + 1$$

$$= \underline{\underline{1681}}.$$

/ 4.3

4.3 Square Roots. The square root of a given number is that number whose square is equal to the given number, for example -

$$2^2 = 2 \times 2 = 4,$$

therefore 2 is the square root of 4.

Similarly, 3 is the square root of 9,
4 is the square root of 16, and so on.

When the sign $\sqrt[2]{\;}$ is placed before a number, for example -

$$\sqrt[2]{25}$$

it indicates that the square root of the number, 25 in this case, is to be found.

In practice, when dealing with the square root, the index ^{2} is usually omitted, for example -

$\sqrt{25}$ indicates the square root of 25.

The sign $\sqrt{\;}$ for square root is simply an r (for root) that has been distorted.

4.4 Finding the Square Root of Numbers. The square roots of some numbers can be found by factorisation, for example -

What is the square root of 225?

$$\begin{aligned} 225 &= 3 \times 3 \times 5 \times 5 \\ &= 15 \times 15 \\ &= 15^2 \end{aligned}$$

Therefore $\sqrt{225} = 15$.

Few numbers can be factorised in this manner and a more usual method of finding the square root is by successive approximation. This method is shown in the following examples -

(i) Find $\sqrt{1156}$.

First Step. Starting from the right hand side, first mark off the figures in pairs as shown -

$$11'56$$

The number of groups, in this case two, signifies the number of figures in the square root, to the left of the decimal point.

Take the first group, 11, and find the nearest perfect square less than 11.

$$3 \times 3 = 9.$$

Three is, therefore, the first figure of the answer. This is the tens figure.

Write this down as follows -

$$\begin{array}{r|l} & \quad 3 \\ \hline & 11'56 \\ 3 \times 3 = & \underline{\;9\;} \end{array}$$

Second Step. Subtract 9 from 11 and bring down the next pair of numbers. Then add a nought to the tens number of the answer already obtained, 3, making it 30, and double it.

$$\begin{array}{r|l} & 3 \\ \cline{2-2} & 11'56 \\ 3 \times 3 = & \underline{9} \\ 30 + 30 = 60 & 256 \end{array}$$

Next determine the largest number, which, when added to 60, and the result multiplied by this same number, will equal or be smaller than 256.

Try 3, then $(60 + 3) \times 3 = 63 \times 3 = 189$,

Try 4, then $(60 + 4) \times 4 = 64 \times 4 = 256$.

$$\begin{array}{r|l} & 3\ \ 4 \\ \cline{2-2} & 11'56 \\ 3 \times 3 = & 9 \\ \cline{2-2} 30 + 30 = 60 & 256 \\ (60 + 4) \times 4 & 256 \\ \cline{2-2} & \end{array}$$

This problem works out evenly. The second figure of the answer is, therefore, 4. This is the units figure.

In practice, most of the above reasoning is done in the head -

$$\begin{array}{r|l} & 3\ \ 4 \\ \cline{2-2} 3 & 11'56 \\ & 9 \\ \cline{2-2} 64 & 256 \\ & 256 \\ \cline{2-2} & \end{array}$$

Therefore $\sqrt{1156} = 34$.

When the problem contains more than two groups of figures, additional steps are necessary. Each subsequent step is similar to the second step -

(i) Bring down the remainder and the next pair of figures to the next line.

(ii) Add a nought to the number already obtained in the root and double it.

(iii) Find the largest number which added to (ii) above and the result multiplied by the same number is equal to or smaller than (i) above. This number is included as part of the answer.

4.5 Square Roots of Decimal Numbers.

Find $\sqrt{118.375}$.

When dealing with decimal numbers the figures are marked off in pairs, starting at the decimal point and working left and right -

1'18.37'50'.

When the number of decimal figures is uneven, a nought must be added to make an even number of figures on the right of the decimal point.

```
         1  0. 8  8
    1  | 1'18.37'50
       | 1
  208  |   1837
       |   1664
 2168  |    17350
       |    17344
       |        6
```

Therefore $\sqrt{118.375} = 10.88.$

It is important to correctly place the decimal point in the answer. This is placed directly above the decimal point in the problem.

Remember - The number of groups of figures to the left of the decimal point in the problem also signifies the number of figures to the left of the decimal point in the answer.

In some numbers, the process of finding the square root may be carried on indefinitely, and it will never exactly finish without a remainder. Usually in this case, two or three decimal places will be sufficiently accurate in the answer. For example -

Find $\sqrt{2}.$

```
         1. 4  1  4
    1  | 2.00'00'00'
       | 1
   24  | 100
       |  96
  281  |   400
       |   281
 2824  |   11900
       |   11296
       |     604
```

Therefore $\sqrt{2} = 1.414$ approximately.

4.6 Square Roots of Fractions. To determine the square root of a fraction, it is necessary to find the square root of the numerator and of the denominator separately.

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}.$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}.$$

To find the square root of a mixed fraction, reduce the expression to an improper fraction and then proceed as before.

$$\sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} = 1\frac{1}{2}.$$

$$\sqrt{5\frac{1}{16}} = \sqrt{\frac{81}{16}} = \frac{9}{4} = 2\frac{1}{4}.$$

When a fraction cannot be reduced to a square root by this method, the fraction is converted to a decimal and the square root of the decimal number is found.

5. RATIO AND PROPORTION.

5.1 Simple Equations. The student should understand the following before considering ratio and proportion.

Consider the statement -

$$4 + 2 = 6.$$

This is known as an equation. An equation is simply a statement of truth and the truth is maintained only when both sides of the equation are treated alike. For example, when we add a certain number to one side of the equation, we must also add the same number to the other side of the equation, otherwise the resulting equation is not true -

$$(4 + 2) + 3 = 6 + 3.$$

Similarly, when we subtract some number from one side of the equation, or multiply or divide it by some number, we must do the same operation to the other side to maintain the truth of the equation -

$$(4 + 2) - 3 = 6 - 3.$$
$$(4 + 2) \times 3 = 6 \times 3.$$
$$(4 + 2) \div 3 = 6 \div 3.$$

Also, we can square or take the square root of each side of the equation without upsetting the balance -

$$(4 + 2)^2 = 6^2.$$
$$\sqrt{4 + 2} = \sqrt{6}.$$

Now consider an equation of the type -

$$\frac{2}{4} = \frac{6}{12}.$$

Similar reasoning applies in this case. For example, when we multiply both sides by the two denominators -

$$\frac{2}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{1} \times \frac{12}{1} = \frac{6}{\underset{1}{\cancel{12}}} \times \frac{4}{1} \times \frac{\overset{1}{\cancel{12}}}{1}$$

cancelling, $2 \times 12 = 6 \times 4.$

Note that the cross products of the original equation are equal. This is termed cross multiplication.

$$\frac{2}{4} \times \frac{6}{12}$$
$$24 = 24.$$

5.2 Ratio. A ratio is defined as the relationship which exists between two quantities of the same kind, and signifies the number of times one is contained in the other. For example, when a comparison is made between two objects and one is said to be twice as big as the other, it is called a ratio. "Twice as big as" means a ratio of "two to one".

When we compare the lengths of a 12' pole and a 36' pole, the ratio is 12 to 36, which is the same as 1 to 3. This is generally written 1 : 3.
When we compare the lengths of a 36' pole and a 12' pole, the ratio is 3 : 1.
A ratio can also be expressed as a fraction.
The 12' pole is 1/3 times the length of the 36' pole.
The 36' pole is 3/1 times the length of the 12' pole.

When dealing with ratios in this manner, both terms must be of the same kind. In the above case they are both expressed in feet.

/ Ratios

Ratios are also used to indicate the composition of materials, for example -

In the mixing of concrete, the mixture 1 : 2 : 4 indicates the ratio by measurement of cement, sand and metal.

Ratios are sometimes expressed in percentage, for example -

Tar and tallow compound used for greasing bolts and coach screws is 25% tar : 75% tallow.

Sometimes the % sign is omitted, for example -

Wiping solder 35/65 indicates the composition of tin and lead in the solder -

35% tin : 65% lead.

Similarly, solder 50/50 indicates 50% tin : 50% lead.

5.3 Proportion. When two ratios are equal to one another, the four terms are said to be in proportion.

Thus 4 : 2 :: 18 : 9 represents two ratios 4 : 2 and 18 : 9 which are equal to one another. The expression is read as follows -

4 is to 2 as 18 is to 9.

The outer terms, 4 and 9, are called the extremes.

The inner terms, 2 and 18, are called the means.

Using the fractional method, the ratios are written as 4/2 and 18/9.

Since they are equal, $\frac{4}{2} = \frac{18}{9}$.

Cross-multiplying, $4 \times 9 = 18 \times 2$.

Note that the product of the extremes equals the product of the means.

The two terms of a ratio must be of the same kind, but it is not necessary that the two ratios of a proportion are of the same kind. This is seen in the following problems.

5.4 Direct and Inverse Proportion. A knowledge of proportion is necessary for the solution of many problems. A proportion has four parts, but when we know any three, we can always find the fourth. The solution is based on the fact that the cross products of an equation are equal (see paragraph 5.1).

There are two kinds of proportions - Direct and Inverse.

In a direct proportion, as one quantity increases the other increases. Similarly, as one quantity decreases the other decreases.

The weight of 1 mile of copper wire is, say, 100 lbs.

The weight of 10 miles of the same wire is, therefore, 100 × 10 = 1000 lbs.

That is, an increase of 10 times in the length gives a corresponding increase of 10 times in the weight. The weight of the wire is directly proportional to its length.

In an inverse proportion, as one quantity increases the other decreases. Similarly, as one quantity decreases, the other increases.

If 1 man can dig a trench in 12 days, how long will 6 men take?

More men engaged on a given task will finish quicker than a fewer number. An increase of 6 times in the number of men reduces the time taken to 1/6 of 12 days = 2 days. The time taken is inversely proportional to the number of men engaged on the job.

/ 5.5

5.5 Problems on Proportion.

Problem No. 1. If it takes a group of men 44 hours to pull in 8 lengths of cable, how long does it take the men to pull in 11 lengths, under the same conditions? It is assumed that the lengths are all equal.

First determine whether this is an example of direct or inverse proportion. More lengths of cable require more time to pull in - a direct proportion.

Next, arrange the facts as a proportion. Then cross-multiply and solve for the unknown quantity.

44 hours : ? hours :: 8 lengths : 11 lengths.

$$\text{or} \quad \frac{44}{?} = \frac{8}{11}$$

Cross multiplying, 8 × Answer in hours = 44 × 11.

To get the answer in hours alone on one side of the equation, we now divide each side of the equation by 8. This does not alter the truth of the equation.

$$\frac{\overset{1}{\cancel{8}} \times \text{Answer in hours}}{\underset{1}{\cancel{8}}} = \frac{44 \times 11}{8}$$

$$\text{Therefore, answer in hours} = \frac{\overset{11}{\cancel{44}} \times 11}{\underset{2}{\cancel{8}}}$$

$$= \frac{121}{2}$$

Answer = 60-1/2 hours.

Problem No. 2. If a certain job takes 9 men 28 days, how long does it take 24 men, working at the same rate, to do the same job?

There are more men on the job, so it takes less time to do. This is an inverse proportion.

If this were a direct proportion, we would set the problem thus -

$$\frac{9 \text{ men}}{24 \text{ men}} = \frac{28 \text{ days}}{? \text{ days.}}$$

But we know the ratio is inverse, so we must invert one of the ratios, thus -

$$\frac{9 \text{ men}}{24 \text{ men}} = \frac{? \text{ days}}{28 \text{ days.}}$$

Cross multiplying -

$$\text{Answer in days} \times 24 = 9 \times 28.$$

Divide each side of the equation by 24 -

$$\text{Answer in days} = \frac{9 \times 28}{24}$$

$$= \frac{\overset{3}{\cancel{9}} \times \overset{7}{\cancel{28}}}{\underset{\underset{2}{\cancel{8}}}{\cancel{24}}}$$

$$= \frac{21}{2}$$

Answer = 10-1/2 days.

/ Problem

Problem No. 3. In each of the two previous examples, a certain statement was made, and the result of altering one term was found. When, at the same time, another variable factor is introduced, there are now two variables and the proportion is said to be compound. A compound proportion consists of a series of simple proportions, either direct or inverse.

If 4 men erect 5 poles in 10 hours, how long does it take 8 men to erect 15 poles?

First, let us see what we are comparing -

First job	10 hours	5 poles	4 men
Second job	? hours	15 poles	8 men.

The fraction with the unknown in it is placed at the left of the equals sign, because it is what we are trying to find. We use it just as it appears in the problem -

$$\frac{10}{?\text{ hours}} =$$

Now set up the first part of the proportion. This is a direct proportion - more poles take more time - so the order is the same as in the chart.

$$\frac{10}{?\text{ hours}} = \frac{5}{15}.$$

Then set up the last part of the proportion. This is an inverse proportion - more men take less time - so the figures from the chart are inverted or turned over.

$$\frac{10}{?\text{ hours}} = \frac{5}{15} \times \frac{8}{4}$$

$$\frac{10}{\text{Answer in hours}} = \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}}} = \frac{2}{3}.$$

Cross multiplying -

$$\text{Answer in hours} \times 2 = 10 \times 3.$$

Dividing each side by 2, -

$$\text{Answer} = \frac{10 \times 3}{2} = \frac{30}{2}$$

$$= \underline{\underline{15\text{ hours}}}.$$

Thus a compound proportion is the same as a simple proportion with one extra step of multiplying the ratios to the right of the "equals" sign.

Problem No. 4. The weight of a coil of 200 lb. H.D.C. wire is 85 lbs. What is its length?

200 lb. H.D.C. signifies the weight per mile, that is 200 lbs. per 1760 yards.

Less weight means less length - a direct proportion.

$$200\text{ lbs.} : 85\text{ lbs.} :: 1760\text{ yards} : ?\text{ yards}.$$

$$200 \times \text{Length in yards} = 85 \times 1760$$

$$\text{Length in yards} = \frac{85}{200} \times \frac{1760}{1}$$

$$= \frac{\overset{17}{\cancel{85}}}{\underset{\underset{1}{\cancel{40}}}{\cancel{200}}} \times \frac{\overset{44}{\cancel{1760}}}{1}$$

$$= 17 \times 44$$

$$\text{Answer} = \underline{\underline{748\text{ yards}}}.$$

/ 6.

6. TEST QUESTIONS.

1. Twenty poles are erected on an aerial line route. 75% of these poles have 8 insulators each and the remainder have 6 insulators each. How many insulators are used?

2. Prepare a table showing the squares and square roots of the whole numbers from 1 to 10 inclusive.

3. State briefly what is meant by -

   (i) direct proportion

   (ii) inverse proportion, and

   (iii) compound proportion.

4. One gallon of water weighs 10 lbs. What is the weight of 2-1/2 pints?

5. What is the weight of 100 yards of 100 lb. H.D.C. wire?

6. A drum of kerosene is 11" high and holds 4 gallons when full. How much kerosene is in the drum when the height of the kerosene is 4-1/8"?

7. The weight of 100 yards of cable P.I.Q.L. 1400 pr. /6-1/2 lb. is 23.7 cwt. What is the weight of this cable in tons per mile?

END OF PAPER.

COMMONWEALTH OF AUSTRALIA.

Engineering Branch,
Postmaster-General's Department,
Treasury Gardens,
Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

INTRODUCTION TO ALGEBRA.

CONTENTS:

1. WHAT IS ALGEBRA?

1.1 In arithmetic we deal with numbers. We add, subtract, multiply and divide numbers which represent so many poles, insulators, arms, days, etc.

For example, in arithmetic -

4 arms + 5 arms = 9 arms.
8 poles - 2 poles = 6 poles.

1.2 Algebra is a more general form of arithmetic. It uses the same fundamental processes of addition, subtraction, multiplication and division, but instead of applying them to numbers as in arithmetic, symbols or letters are used, which can represent any number.

When we rewrite the above equations by using letters to represent arms and poles, they are said to be expressed algebraically -

$4a + 5a = 9a$. (In this equation, a signifies 1 arm.)
$8p - 2p = 6p$. (In this equation, p signifies 1 pole.)

1.3 Algebra is mainly a system of abbreviations. Everyone tends naturally to use abbreviations, for example, it is more convenient to say -

G.I. Wire, instead of Galvanised Iron Wire, and
P.I.Q.L. Cable, instead of Paper Insulated Quad Local Cable.

It conveys the essential idea and saves a lot of time. The desire to avoid unnecessary writing was one of the reasons which led to algebra in its modern form.

2. EQUATIONS.

2.1 Problems in algebra are generally solved by the use of equations (see Section 5 of Paper No. 3). Equations are merely statements of fact. When a sentence states that -

$$\text{something}\left\{\begin{array}{l}\text{is}\\ \text{equals}\\ \text{is the same as}\end{array}\right\}\text{something else,}$$

it is an equation.

2.2 Equations can be written in shorthand form. For example, consider the statement -

"The height of the pole is 25 feet."

When we substitute the letter H for the height of the pole, we rewrite the statement thus -

$$H = 25'.$$

This is an algebraic equation.

Similarly, the statement -

"The length of the arm is 80 inches" is rewritten as -

$$L = 80,$$

where L signifies the length of the arm in inches.

The rewriting of equations in algebraic form saves using a lot of words, which are awkward in an equation.

2.3 In algebraic equations, any capital or lower case letter of the alphabet is used to represent quantities. It is not necessary to always use letters which correspond to the first letter of the word or phrase they represent. Quite frequently, the letter x is used to signify an unknown quantity.

2.4 Remember: When using an equation -

(i) Equations always balance.

(ii) We must treat each side of the equation the same way for it to remain in balance. (See paragraph 5.1 of Paper No. 3.)

3. SOLUTION OF SIMPLE PROBLEMS BY ALGEBRA.

3.1 The first step in solving simple problems by algebra is to set up the equation, by substituting a letter of the alphabet, say x, for the value we want to find.

It is then necessary to get x alone on the left-hand side of the equation before we can find its value. This is done by applying the various fundamental processes of arithmetic to both sides of the equation.

/ 3.2

3.2 Here are two simple problems showing the use of algebra for their solution.

Problem No. 1. 791 insulators are divided between three jobs in the ratio 1 : 2 : 4. How many insulators are delivered to each job?

Let x = number of insulators delivered to first job,

then $2x$ = number of insulators delivered to second job,

and $4x$ = number of insulators delivered to third job.

Therefore, $x + 2x + 4x = 791$

$7x = 791.$

Divide each side of the equation by 7 -

$x = 113.$

Insulators delivered to first job = 113.

Insulators delivered to second job = $2x$

$= 2 \times 113$

$= 226.$

Insulators delivered to third job = $4x$

$= 4 \times 113$

$= 452.$

Answer = 113, 226 and 452 insulators.

Problem No. 2. 42 poles are distributed between three jobs. 16 poles are delivered to job No. 1 and the remainder shared evenly between jobs Nos. 2 and 3. What is the allocation to each of these jobs?

Let x = number of poles delivered to each of the jobs Nos. 2 and 3.

Therefore, $2x$ = total number of poles delivered to jobs Nos. 2 and 3.

$2x + 16$ = total number of poles distributed.

$2x + 16 = 42.$

Subtract 16 from each side of the equation -

$2x + 16 - 16 = 42 - 16$

$2x = 26.$

Divide each side of the equation by 2 -

$$\frac{2x}{2} = \frac{26}{2}$$

$x = 13.$

Answer = 13 poles are delivered to each of the jobs Nos. 2 and 3.

/ 4.

4. FORMULAE.

4.1 A formula is the statement of a general truth expressed in abbreviated form, usually by letters. It is a statement that covers many similar situations.

4.2 Consider, for example, this statement which is known as Ohms Law -

> "The current in amperes flowing in an electrical circuit is directly proportional to the electromotive force applied to the circuit and inversely proportional to the resistance in ohms of the circuit."

In abbreviated form, we could say -

> "The current in amperes is directly proportional to the e.m.f. in volts and inversely proportion to the resistance in ohms."

This could be shortened still further to -

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}},$$

and finally to -

$$I = \frac{E}{R},$$

where I is the current in amperes,
E is the e.m.f. in volts,
R is the resistance in ohms.

This final equation is the form used in modern algebra and represents the original statement expressed in algebraic form.

4.3 A rule expressed by letters such as $I = \frac{E}{R}$ is known as a formula. The plural of formula is often formulae. Formula is a Latin word, and the Latin plural has become part of the English language. Some formulae are found by experiment, others purely by reasoning. The development and proof of formulae, however, are beyond the scope of a course of this nature.

4.4 Similarly, the following statement -

> "The tension in pounds on a stay is directly proportional to the product of the length of the stay in feet and the total tension in pounds (sometimes called the horizontal stress) of the wires on the pole, and is inversely proportional to the distance in feet from the pole to where the stay emerges from the ground",

is expressed as a formula, thus -

$$T = \frac{L \times W}{D},$$

where T = tension on stay (in lbs.),

L = length of stay (in feet),

W = total tension of wires (in lbs.),

D = distance from pole to point of emergence of stay (in feet).

This formula is also written as -

$$T = \frac{LW}{D}.$$

/ 4.5

4.5 In a formula, when two letters are put next to each other without a sign, multiplication is always understood. The reason for this is that the multiplication sign, ×, looks very much like the letter x, and might be mistaken for it. So, in algebra, multiplication signs are often left out. Similarly, a letter or figure before a bracket also signifies multiplication, as in arithmetic, even though the sign is omitted between the letter or figure and the bracket. (See paragraph 2.4 of Paper No. 3.)

4.6 The two examples just given show how we can make up our own formula. We must first understand the method by which a certain type of problem is solved, we then put that method in the form of a general rule, and then bring in abbreviations. Other formulae are given in Papers Nos. 5 and 6.

5. USING FORMULAE.

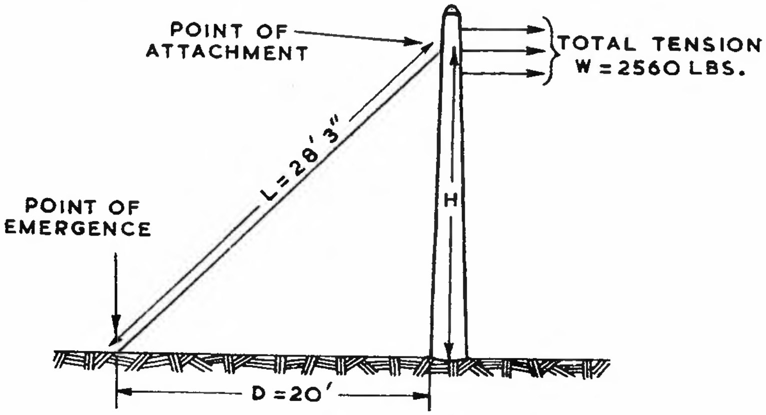
5.1 Now let us see how to use a formula by taking, for example, the formula given in paragraph 4.4 -

$$T = \frac{L \times W}{D}.$$

This formula shows us how to calculate the tension on the stay (T), when we know the length of the stay (L), the total tension on the arms (W), and the distance from the pole to the point of emergence of the stay (D).

5.2 In practice, a problem of this type may arise (see Fig. 1.) -

Problem. The total tension on the arms of a terminal pole due to the line wires is 2,560 lbs. A terminal stay 28 feet 3 ins. long is attached to the pole. The distance from the pole to the point of emergence of the stay is 20 feet. What is the tension on the stay?



CALCULATING TENSION ON STAY.

FIG. 1.

To apply the formula to this problem, we rewrite the formula, substituting numbers for the corresponding letters, thus -

$$T = \frac{L \times W}{D}.$$

Write the length of the stay in feet in this square.

Write the total tension in lbs. in this square.

$$= \frac{\square \times \square}{\square}$$

Write the distance in feet from the pole to point of emergence of stay in this square.

/This

This diagram shows what the formula really means. It tells us what to write down and how to begin the problem. The problem is now solved by simple arithmetic -

$$T = \frac{28\frac{1}{4} \times 2560}{20}$$

$$= \frac{113 \times \overset{\overset{32}{\cancel{640}}}{\cancel{2560}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{20}}}$$

$$\begin{array}{r} 113 \\ \underline{32} \\ 226 \\ \underline{3390} \\ 3616 \end{array}$$

$$= 113 \times 32$$

$$= 3616.$$

Answer = <u>3,616 lbs.</u> tension on the stay.

5.3 Another simple problem in the use of a formula is shown.

<u>Problem.</u> Convert a temperature of 86 degrees Fahrenheit to the equivalent value on the Centigrade scale.

Degrees Centigrade (C) are found from degrees Fahrenheit (F) by using the formula -

$$C = \frac{5(F - 32)}{9}.$$

Substituting the value given above for F, in the formula -

$$C = \frac{5(86 - 32)}{9}$$

$$= \frac{5 \times 54}{9}$$

$$= \frac{5 \times \overset{6}{\cancel{54}}}{\underset{1}{\cancel{9}}}$$

$$= 5 \times 6$$

$$= 30.$$

Answer = <u>30 degrees Centigrade</u>.

To prove this answer, substitute 30 for C in the above formula and solve for F.

$$30 = \frac{5(F - 32)}{9}.$$

(Multiply both sides by 9,) $270 = 5(F - 32)$

$$270 = 5F - 160.$$

(Add 160 to each side,) $270 + 160 = 5F - 160 + 160$

$$430 = 5F.$$

(Divide each side by 5,) $86 = F$,

which agrees with the original value for F in the problem.

6. TRANSFORMATION OF FORMULAE.

6.1 Sometimes, the formula which suits our problem is not given in the most convenient form.

Ohms law formula (see paragraph 4.2), $I = \frac{E}{R}$, shows how to find the current when we know the e.m.f. and resistance of the circuit.

But suppose we want to find E instead of I. How can we use this formula? We use the same rules as for solving equations (see Section 3). We rearrange the formula so as to put E alone on one side just as we rearranged the equation to place x alone on one side.

In the formula $I = \frac{E}{R}$, E is being divided by R.

When we do the opposite, that is, multiply each side of the formula by R, we do not alter the truth of the formula -

$$I \times R = \frac{E}{R} \times R$$

$$I \times R = \frac{E}{\underset{1}{\cancel{R}}} \times \frac{\overset{1}{\cancel{R}}}{1}$$

$$I \times R = E.$$

$$\text{Therefore,}\quad E = I \times R$$

$$= IR \text{ (omitting the multiplication sign).}$$

$$\text{Similarly,}\quad R = \frac{E}{I}$$

Any formula can be rearranged to give any one of the letter values.

6.2 As another typical example, consider the formula -

$$T = \frac{L \times W}{D} \quad \text{(see paragraph 4.4).}$$

Suppose we know the tension (T) which the stay withstands, and wish to find the total tension (W) on the arms which produces this value of stay tension. This case may arise when we add line wires to an existing route and want to know whether the existing stay will carry the additional tension of the wires.

In this case, we want W alone on one side of the formula. W is being multiplied by $\frac{L}{D}$. Therefore divide each side of the formula by $\frac{L}{D}$, that is, multiply by $\frac{D}{L}$, thus -

$$T \times \frac{D}{L} = \frac{L \times W}{D} \times \frac{D}{L}$$

$$T \times \frac{D}{L} = \frac{\overset{1}{\cancel{L}} \times W}{\underset{1}{\cancel{D}}} \times \frac{\overset{1}{\cancel{D}}}{\underset{1}{\cancel{L}}}$$

$$\frac{T}{1} \times \frac{D}{L} = \frac{W}{1}.$$

$$\text{Therefore,}\quad W = \frac{T \times D}{L}.$$

Alternative solution -

$$T = \frac{L \times W}{D}.$$

Cross multiplying, $L \times W = T \times D$.

Divide each side by L,

$$\frac{L \times W}{L} = \frac{T \times D}{L}.$$

$$\text{Therefore,}\quad W = \frac{T \times D}{L}.$$

7. CONVERTING FORMULAE INTO PICTURES.

7.1 To simplify the working of problems, it is convenient in many cases to convert the formula into a graph from which the answer can be read at a glance. This reduces the amount of calculations when a number of problems of a similar type are to be solved.

7.2 Constructing a Graph. To take a simple example, consider the formula -

$$T = \frac{L \times W}{D} \quad \text{(see paragraph 4.4).}$$

Assuming fixed values for L and D, it is possible to show by a graph how the value of T varies according to the value of W.

Suppose the values of L and D are fixed so that the ratio $\frac{L}{D} = 1.414$. (This ratio exists when the distance, D, from the pole to the point of emergence of the stay equals the height of the pole above ground to the point of attachment of the stay. This height is designated H in Fig. 1.) Then the formula may be rewritten -

$$T = 1.414 \times W.$$

By substituting different values for W, the corresponding values for T are calculated from this formula. The results are summarised in table form, thus -

Value of W (in lbs.)	Value of T (in lbs.)	
0	0	
1000	1414	(Point A on graph.)
2000	2828	(Point B on graph.)
3000	4242	(Point C on graph.)
4000	5656	(Point D on graph.)

This table is continued for as long as we like. From this table, the graph is constructed by setting out two lines at right angles, (see Section 4 of Paper No. 5) as shown in Fig. 2.

On one of these lines (it does not matter which), we set out to a convenient scale, measurements representing one set of quantities, and on the other line, also to a convenient scale, measurements representing the other set of quantities.

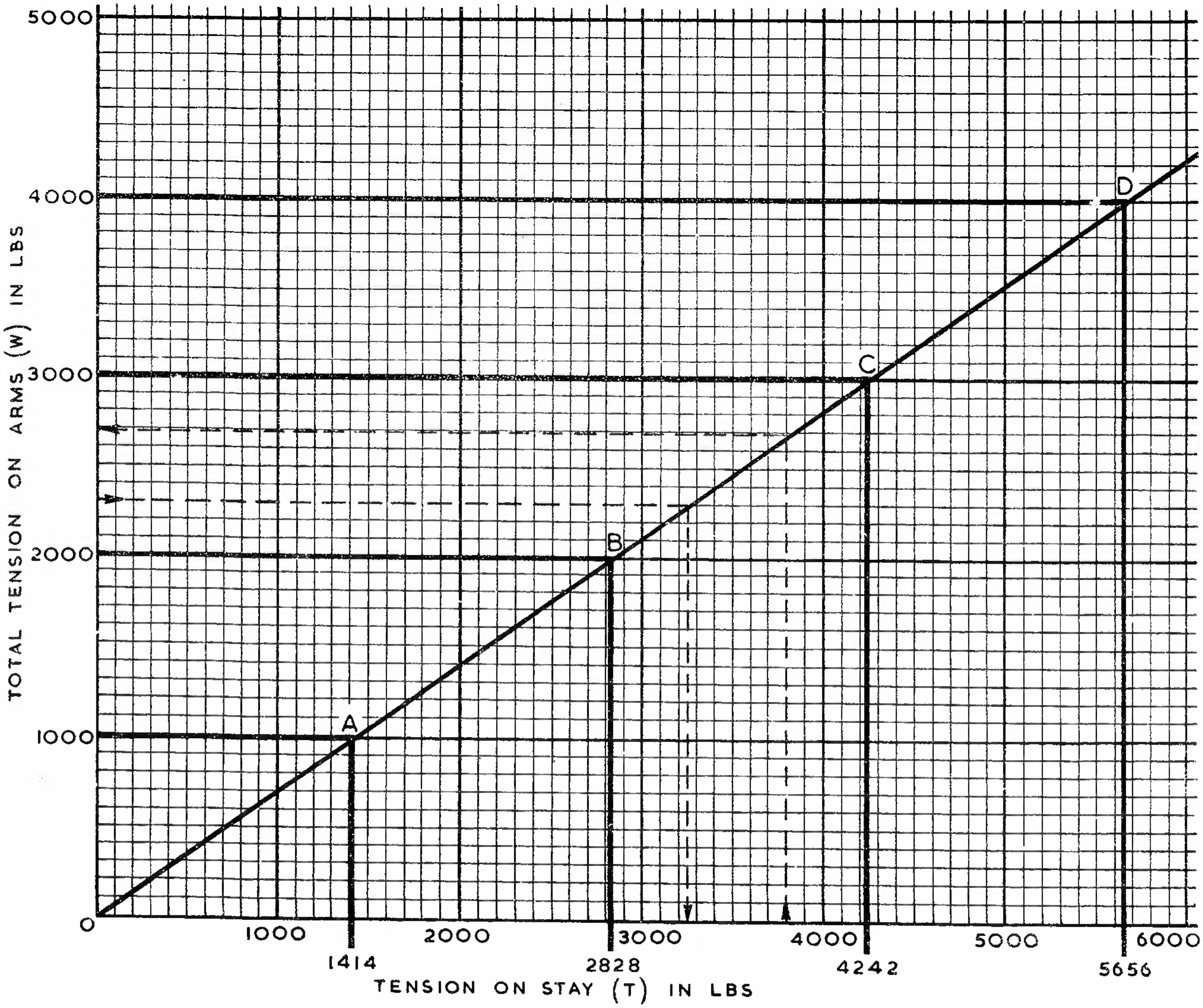
In the example chosen, we set out values of W on the vertical scale and values of T on the horizontal scale. On both scales, the values are written as shown in Fig. 2. No graph is complete unless these figures are given, with information as to what they represent. The scale chosen in each case is 1" for every 1000 lbs. Other scales could have been chosen, the choice being one of convenience and suitability. On the graph shown, horizontal and vertical lines are drawn at these 1" intervals. Each of these are further subdivided in 10 equal divisions. Each small division, therefore, represents 100 lbs.

Now from point 1000 on the W scale and point 1414 (just a fraction over 1400) on the T scale, horizontal and vertical lines are drawn. These meet at point A in Fig. 2. Similarly, horizontal and vertical lines are drawn from the W and T scales, respectively, for the other readings given in the table. When a line is drawn through these points, the result is a graph from which we can find -

(i) the tension on the stay for any value of total tension on the arms up to about 4000 lbs., and

(ii) the total tension permitted on the arms for a particular value of tension on the stay.

/ Fig. 2.



GRAPH OF T = 1.414 × W.

FIG. 2.

7.3 From the graph in Fig. 2, find -

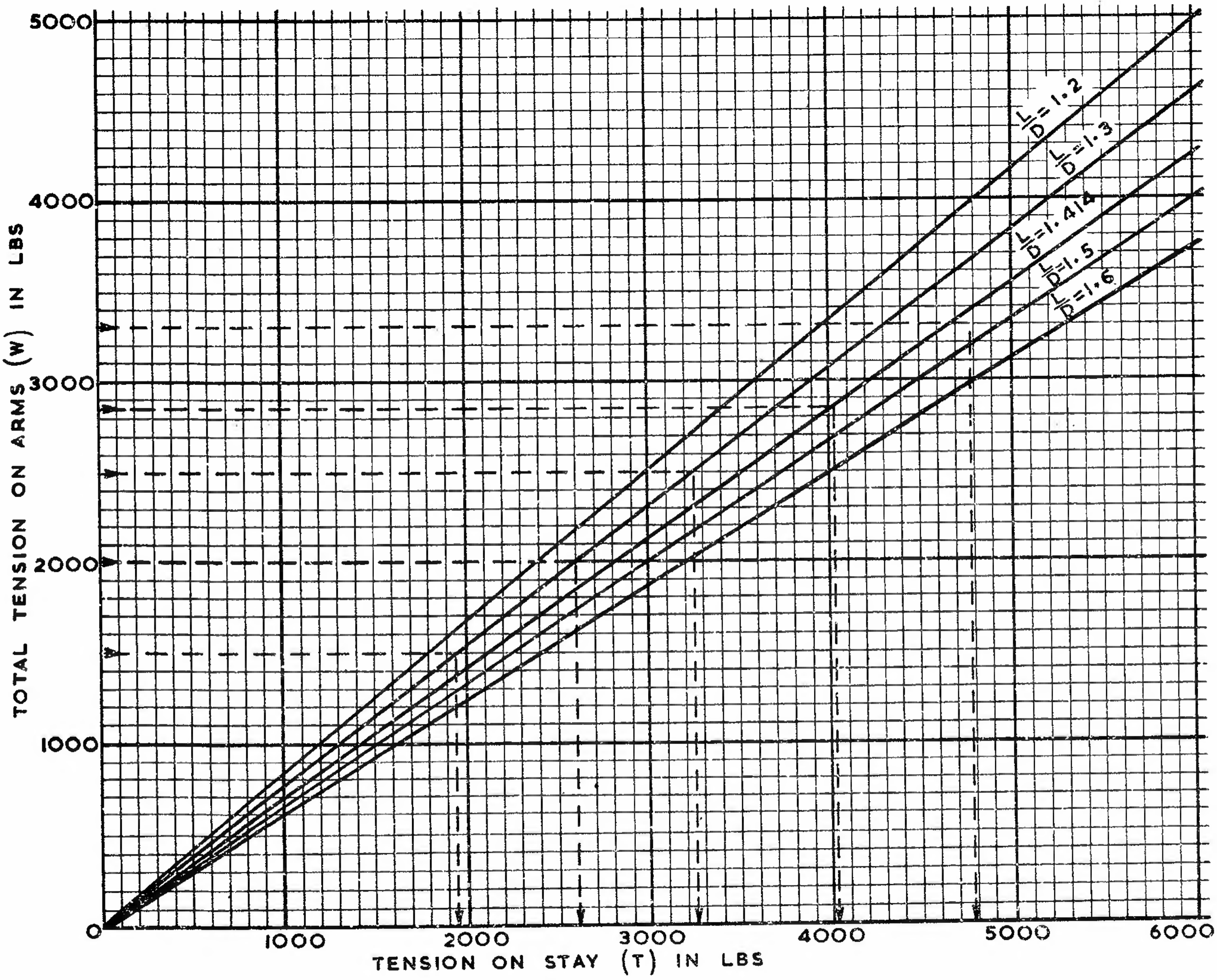
(i) the value of T when W = 2,300 lbs., and

(ii) the value of W when T = 3,800 lbs.

Then calculate the values from the appropriate formulae.

It is important to note that the results obtained from the graph do not always agree exactly with those obtained when we calculate the values. However, the graph results are usually sufficiently accurate for most practical purposes.

7.4 Fig. 2 applies for one particular value of the ratio $\frac{L}{D}$. The usefulness of the graph is increased by including on it, graphs for other values of $\frac{L}{D}$ as shown in Fig. 3. These are prepared in a similar manner as described for Fig. 2.



GRAPHS FOR DIFFERENT $\frac{L}{D}$ RATIOS.

FIG. 3.

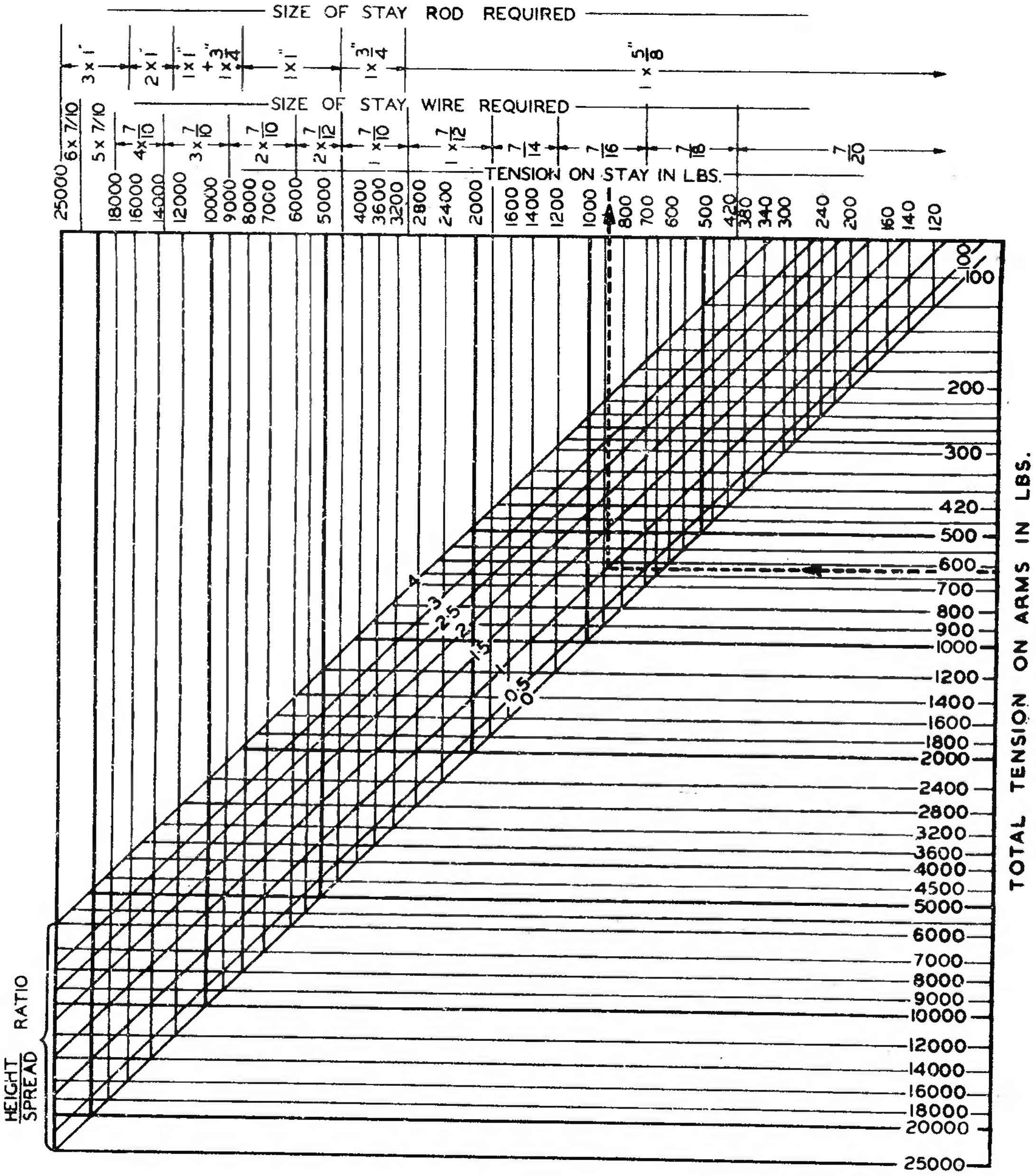
7.5 From the graphs in Fig. 3, find the value of T -

(i) When W = 1,500 lbs. and $\frac{L}{D}$ = 1.3.
(ii) When W = 2,000 lbs., L = 26 ft., D = 20 ft.
(iii) When W = 2,500 lbs., L = 19 ft. 6 ins., D = 15 ft.
(iv) When W = 2,840 lbs., L = 28 ft. 3 ins., D = 20 ft.
(v) When W = 3,300 lbs., L = 29 ft., D = 20 ft.

In the case of (v), there is no graph for $\frac{L}{D}$ = 1.45. In this case it is necessary to guess the approximate position of the graph which would be drawn for this ratio.

/ 7.6

7.6 Simple graphs of this type are used in practice to select the size of stay rod and wire for a terminal pole (see Fig. 4).



GRAPH FOR TERMINAL POLE STAY WIRES AND RODS.

FIG. 4.

These graphs are similar to Fig. 3, but are drawn for values of $\frac{H}{D}$ and not $\frac{L}{D}$.

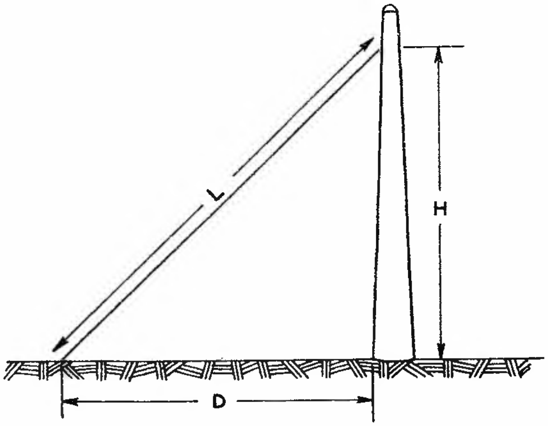
H is the height of the pole from the ground to the point of attachment of the stay. (See Fig. ".)

7.7 The following problem shows the use of the graph in Fig. 4.

Problem. The total tension of the wires on a terminal pole is 625 lbs. The dimension H (see Fig. 5) is 20 feet and the distance D is 20 feet. What is the size of stay wire and stay rod required?

It is first necessary to find the value of $\frac{H}{D}$.

In this case, it is $\frac{20}{20} = 1$. This is called the $\frac{\text{Height}}{\text{Spread}}$ ratio. (An $\frac{H}{D}$ ratio of 1 is the same as an $\frac{L}{D}$ ratio of 1.414 mentioned in paragraph 7.2.)



STAY AT TERMINAL POLE.

FIG. 5.

To use the graph, imagine a horizontal line drawn to correspond to a total tension of 625 lbs. At the point where this horizontal line cuts the graph corresponding to $\frac{H}{D} = 1$, imagine a vertical line. This line indicates from the graph -

(i) the tension on the stay is about 880 lbs.,

(ii) one 7/16 stay wire is required, and

(iii) one 5/8 inch stay rod is required.

8. TEST QUESTIONS.

1. (i) Rearrange the formula $C = \frac{5(F - 32)}{9}$, so as to get F alone on one side.

   (ii) When F = 50, what is the value of C?

   (iii) When C = 50, what is the value of F?

   (iv) Construct a graph to show the relationship between C and F for any value of F up to 212.

2. From Fig. 4, find the sizes of stay wires and rods for total arm tensions of -

   (i) 1,400 lbs.,

   (ii) 2,350 lbs., and

   (iii) 3,330 lbs., when the height-spread ratio is 1 in each case.

3. In the formula $A = \pi r^2$,

   $A = 1{,}386$, and

   $\pi = 3\frac{1}{7}$.

   What is the value of r?

4. In the formula $L = \sqrt{H^2 + D^2}$, what is the value of L when both H and D equal 20?

END OF PAPER.

COMMONWEALTH OF AUSTRALIA.

Engineering Branch,
Postmaster-General's Department,
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Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

GEOMETRY AND MENSURATION.

CONTENTS:

1. INTRODUCTION.

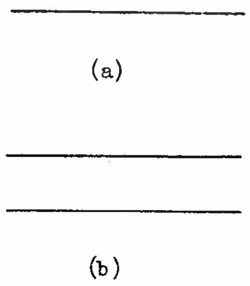
1.1 Geometry is arithmetic - addition, subtraction, multiplication and division - applied to the shape of objects rather than to numbers. Practically all objects can be resolved into shapes based on straight lines and circles, and this study of geometry is concerned with the shape of simple figures composed of lines and circles.

1.2 Mensuration deals with the calculation of areas of these objects.

2. LINES AND ANGLES.

2.1 Straight Lines. A straight line (see Fig. 1a) is a line which maintains a certain direction for its whole length. A tightly stretched rope or wire forms a straight line.

2.2 Parallel Lines. Two lines are parallel to one another when they are the same distance apart for their whole length (see Fig. 1b). When two such lines are continued indefinitely both ways, they do not meet. The two sides of an ordinary flat ruler are straight and parallel to one another.



STRAIGHT AND PARALLEL LINES.

FIG. 1.

2.3 Angles. Fig. 2a shows a number of pairs of lines which are not parallel. These lines do not meet each other, so it is convenient to make the lines of each longer until they touch. (See Fig. 2b.) When two lines meet like this, they are said to form an angle.

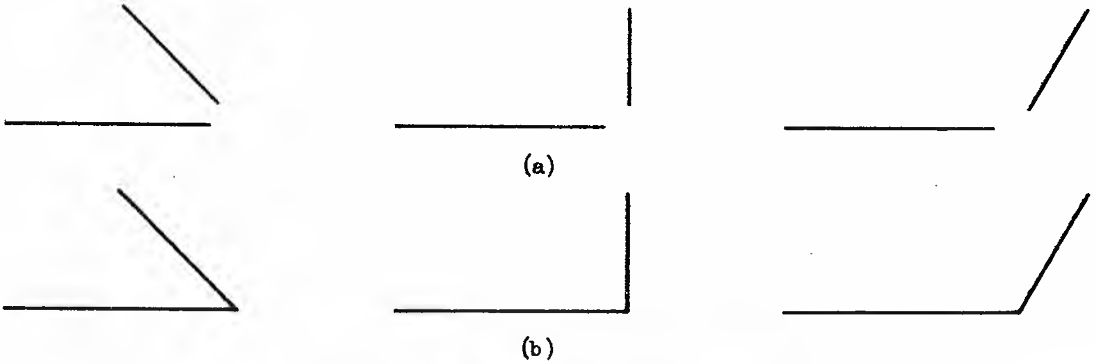
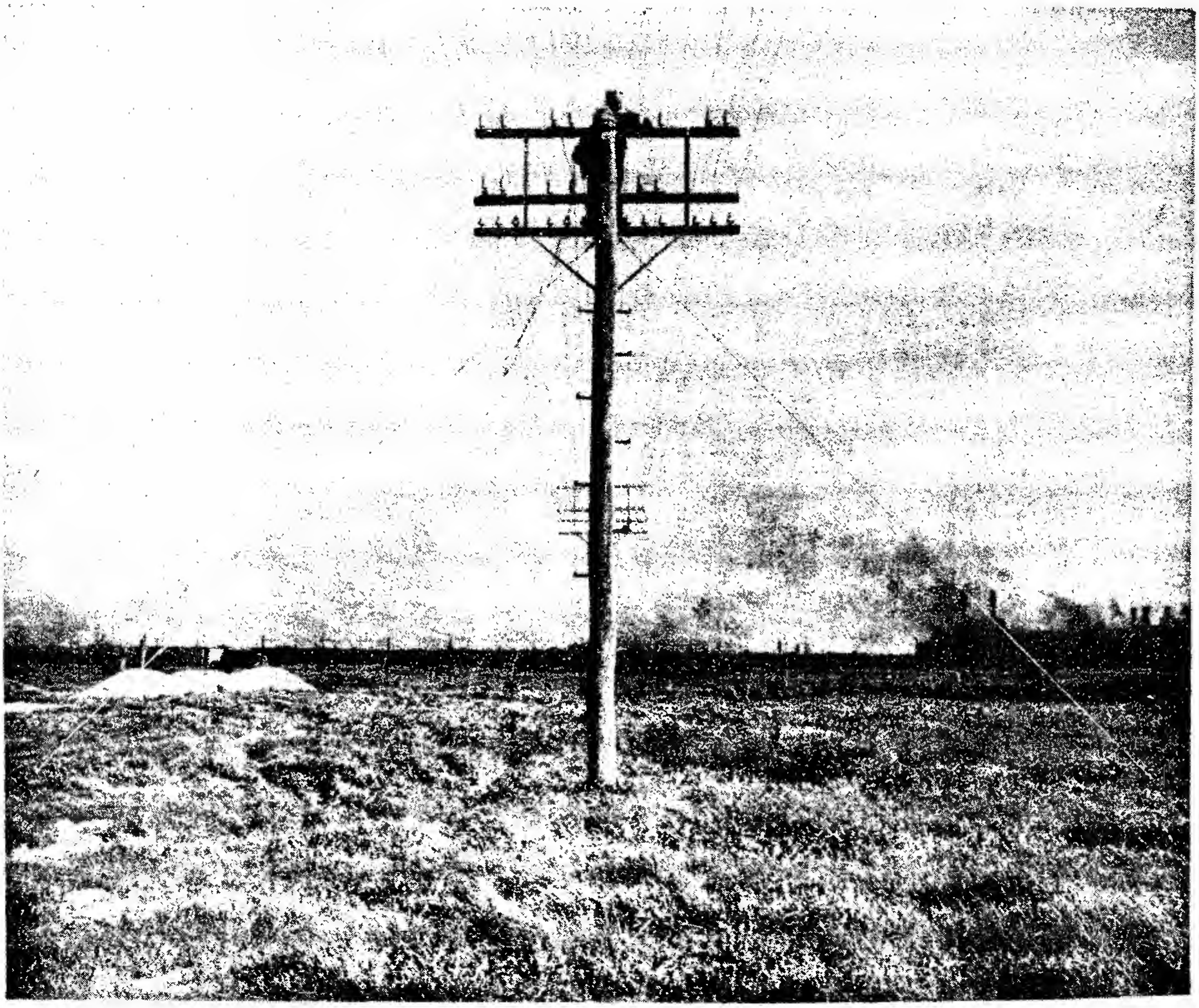


FIG. 2. ANGLES FORMED BY STRAIGHT LINES.

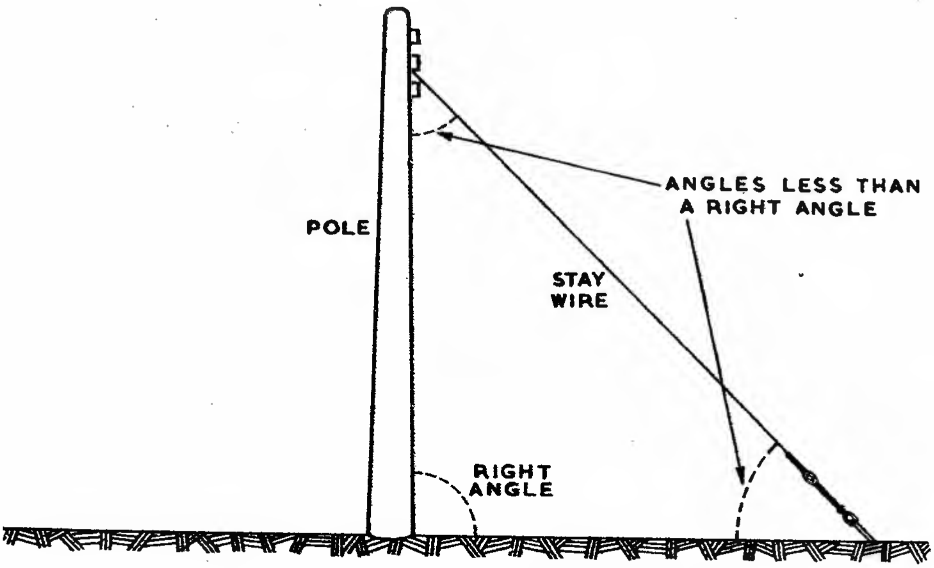


LINES AND ANGLES.

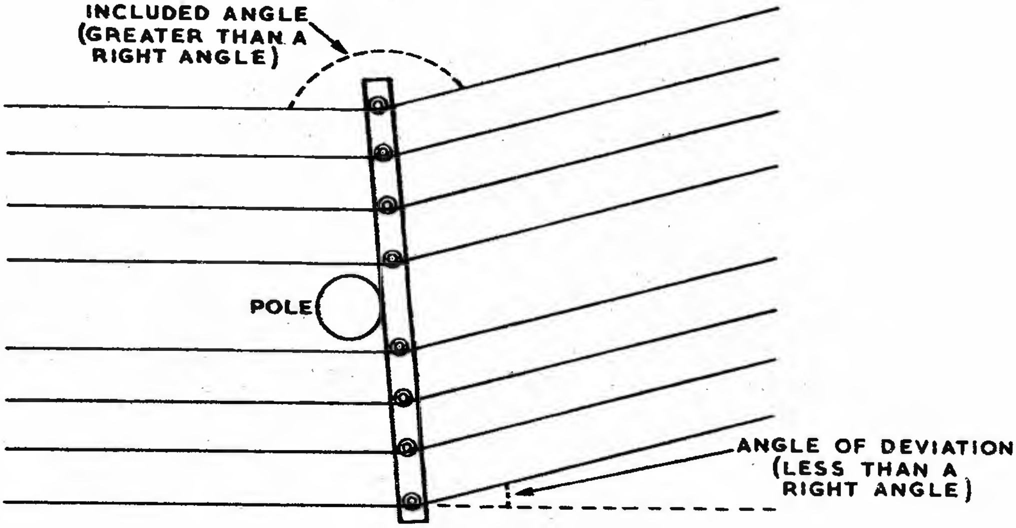
The three angles shown in Fig. 2 are quite different in appearance. The first is something like the angles formed between a stay wire and the ground, and between a stay wire and a pole (see Fig. 3a).

The second which looks like the corner of a book, has a special name. It is called a right angle. When two lines make a right angle, they are said to be perpendicular to each other. A right angle is formed between a pole and the ground when the pole is set in level ground (see Fig. 3a).

The third is something like the angle formed by the line wires on each side of the arm when the route changes direction. This angle is called the included angle (see Fig. 3b).



(a) Terminal Stay.

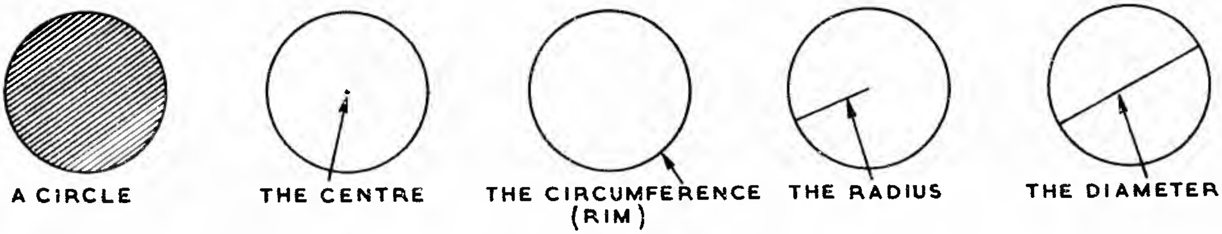


(b) Line Wires at Angle Pole.

FIG. 3. ANGLES IN LINE CONSTRUCTION.

3. CIRCLES.

3.1 A circle (see Fig. 4) is a figure enclosed by a line traced out in such a way that it is always a constant distance from a given point. This point is called the centre of the circle.



TERMS CONNECTED WITH A CIRCLE.

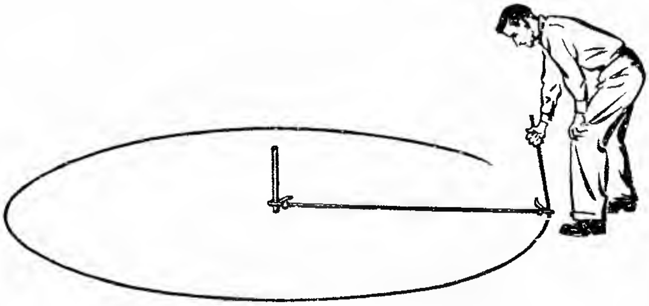
FIG. 4.

The line enclosing the circle is called the circumference or rim.

The line joining any point on the circumference of a circle to the centre is called the radius.

When the radius is continued through the centre to cut the circumference, this distance is called the diameter of the circle. The length of the diameter is twice the length of the radius.

3.2 A circle can be made with two sticks and a length of rope. Fig. 5 shows this being done.



MARKING OUT A CIRCLE.

FIG. 5.

A similar arrangement on a smaller scale could be used when drawing on paper, but usually compasses or dividers are used.

3.3 Many items used in line work are circular in shape, for example -

wooden poles,
bolts,
insulators,
line wires, etc.

/ 3.4

3.4 Relationship between Circumference, Diameter and Radius. The ratio of the length of the circumference to that of the diameter is always a constant value -

$$\frac{\text{circumference}}{\text{diameter}} = \text{constant value.}$$

This is true for any circle. The constant is referred to by the greek letter π, called Pi, and is equal to 3.1416 correct to four decimal places. In problems, π is usually approximated to 3.14 or $\frac{22}{7}$.

$$\frac{\text{Circumference}}{\text{diameter}} = \pi = 3.14 \text{ or } \frac{22}{7}.$$

$$\text{Therefore, circumference} = \pi \times \text{diameter}$$

$$= \pi \times \text{twice radius}$$

$$= 2\pi \times \text{radius.}$$

$$\text{Also, diameter} = \frac{\text{circumference}}{\pi},$$

$$\text{and radius} = \frac{\text{circumference}}{2\pi}.$$

It is important not to confuse the diameter with the radius, and care should be taken in problems to check that any value given is either a radius or a diameter, and to apply the correct formula.

Problem. A length of cable is wound on a cable drum 3'6" in diameter, and forms 21 turns (see Fig. 6). What is the total length of cable?

First find the circumference of the drum. This gives us the length of one turn. Then multiply by the number of turns to give the total length of cable.

Length of cable = circumference of drum × number of turns.

$$= \pi \times \text{diameter of drum} \times \text{number of turns,}$$

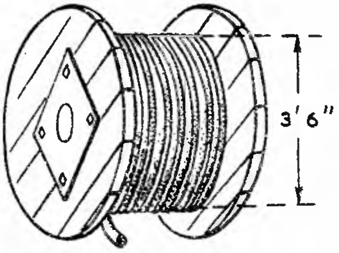
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{21}{1}$$

$$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}} \times \frac{21}{1}$$

$$= 11 \times 21$$

$$= 231 \text{ feet.}$$

$$= \underline{77 \text{ yards}}.$$



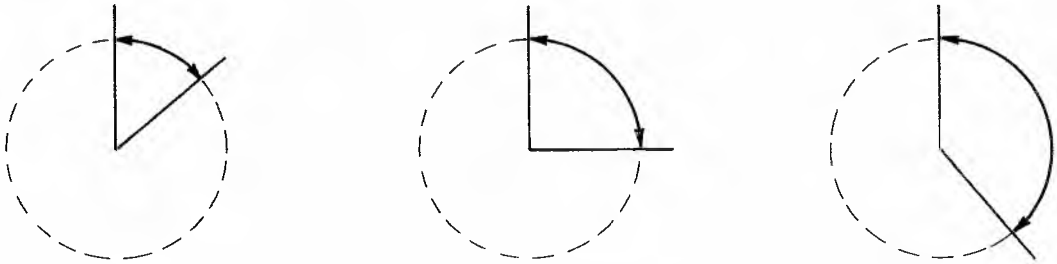
WHAT IS LENGTH OF CABLE?

FIG. 6.

4. MEASUREMENT OF ANGLES.

4.1 It is necessary to have some simple way of measuring angles. There is no simple way to measure an angle in terms of length, but angles are related to several other things that can be measured.

In any circle (see Fig. 7) the lines which form an angle at the centre cut a portion of the circumference. Any angle can, therefore, be imagined to be at the centre of a circle.



ANGLES FORMED AT CENTRE OF CIRCLE.

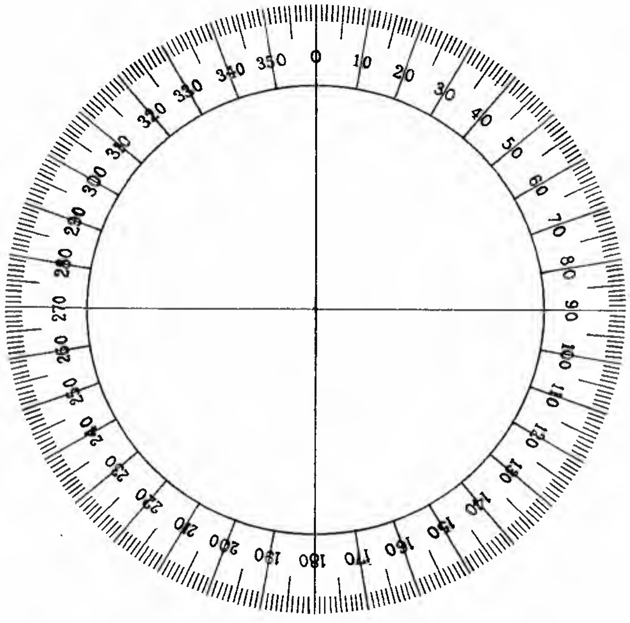
FIG. 7.

In this case, the length of the circumference which is cut by the lines forming the angle is proportional to the size of the angle. Therefore, we could measure the size of the angle by the length of this portion of the circumference.

This method is not satisfactory, however, unless we use a standard circle of specified size. It is more satisfactory to measure the angle by the length of the intercepted circumference, regardless of the size of the circle. This is done by dividing the circumference of the circle into a number of parts and then seeing how many of these parts are enclosed by the lines forming the angle.

4.2 Degrees. In practice, the circumference is divided into 360 equal parts called degrees. The symbol for degrees is °. There are, therefore, 360° in the complete circle. (See Fig. 8.)

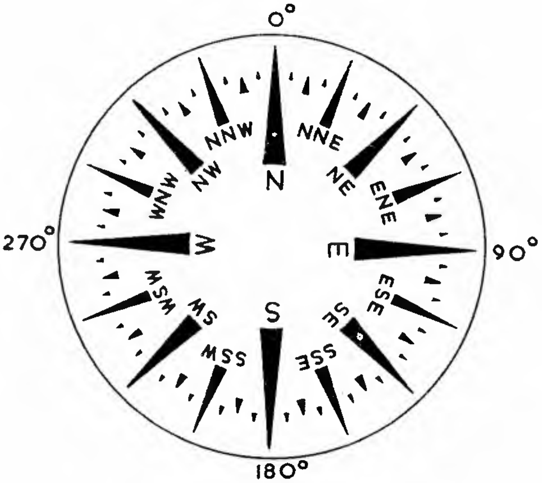
The figure 360 was chosen because it is a number into which many other numbers divide without leaving a remainder. This simplifies calculations when finding a fractional part of a circle or angle.



360° IN A CIRCLE.

FIG. 8.

4.3 Right Angles. In geographical measurements, north is always considered zero degrees (0°). The 360 degrees are then scaled on the circle in a clockwise direction (see Fig. 9).

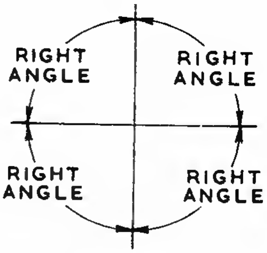


COMPASS.

FIG. 9.

When we start facing north and then turn to the right, we face east. When we turn to the right again, we face south. Again to the right, we face west; once more to the right, we face north.

We have turned to the right four times for the complete circle. We therefore say there are four right angles in a complete circle. (See Fig. 10.)



Each time we turned to the right, we turned through one right angle.

One right angle $= \frac{360^\circ}{4}$

$= 90^\circ$.

360° = 4 RIGHT ANGLES.

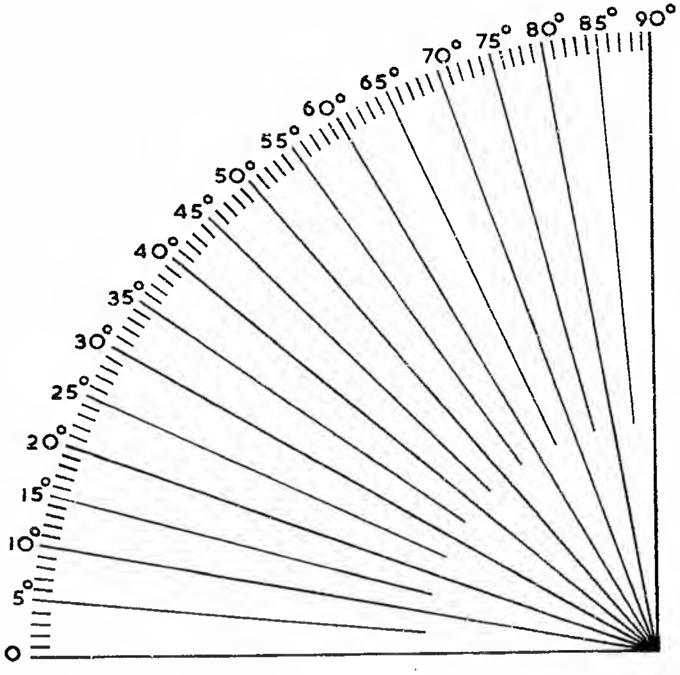
FIG. 10.

Thus, any angle of 90° can also be called a right angle. Some right angles are shown in Fig. 11.

RIGHT ANGLES.

FIG. 11.

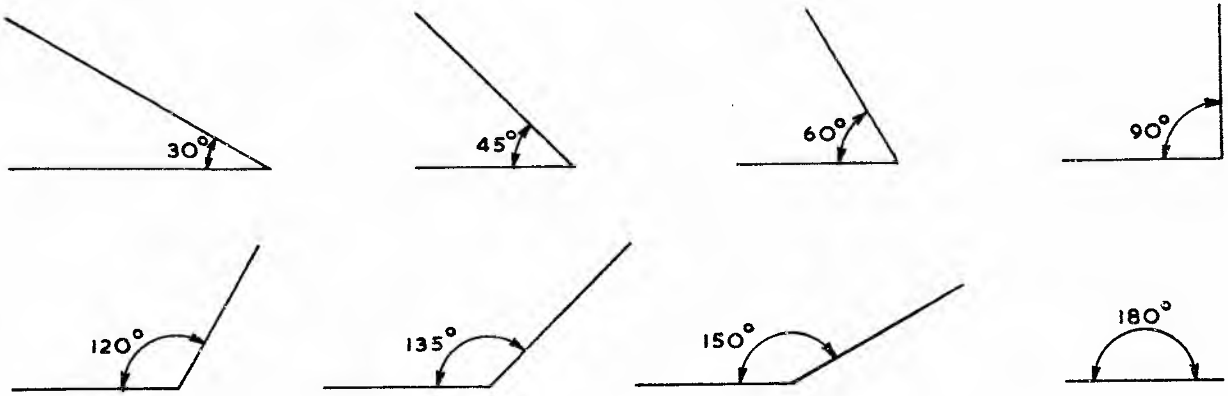
Fig. 12 shows a right angle marked off in intervals of five degrees.



DEGREES IN A RIGHT ANGLE.

FIG. 12.

4.4 Typical Angles. Some typical angles are shown in Fig. 13.



TYPICAL ANGLES.

FIG. 13.

Angles less than 90° are called acute angles. Angles between 90° and 180° are called obtuse angles. When an angle contains two right angles or 180°, the lines forming it are in a straight line.

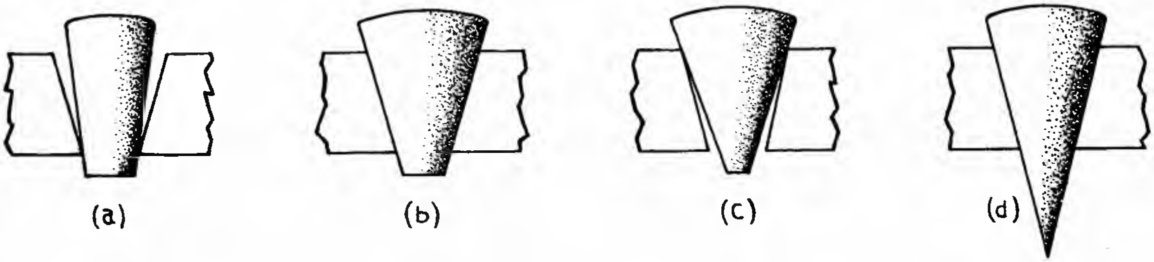
/ In

In some cases angles do not come out an even number of degrees. To meet these cases, a degree is divided into 60 equal parts called minutes. The symbol for minutes is '. Each minute is similarly divided into 60 equal parts called seconds. The symbol for seconds is ". Therefore, an angle is measured in degrees, minutes and seconds. For example, an angle of 45 degrees, 20 minutes, 30 seconds is written -

$$45^{\circ}\ 20'\ 30''.$$

Note. The symbols for minutes and seconds in the measurement of angles are the same as the symbols for feet and inches, respectively, in the measurement of length.

4.5 Importance of Angles. Angles are important when two things have to be fitted together. Figs. 14a, 14b and 14c show three views of stoppers in openings. In Fig. 14a, the angle between the opposite sides of the stopper is less than the angle of the opening. In Fig. 14b, the angle is the same. In Fig. 14c, it is larger. Only the second one gives a correct fit.

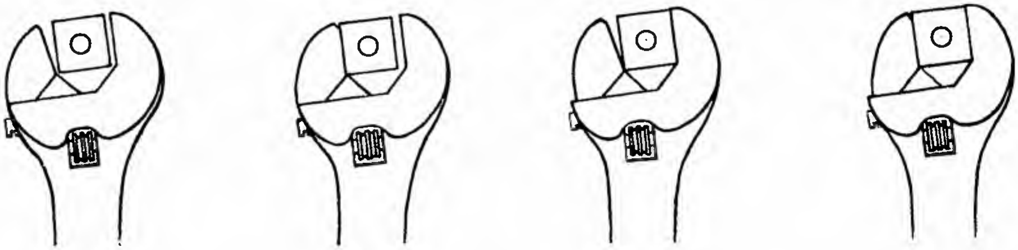


ANGLES ARE IMPORTANT.

FIG. 14.

We can still speak of the angle between the sides of a stopper, although these two lines do not meet. It makes no difference so far as fitting together is concerned, if the sides of the stopper were extended until they meet as shown in Fig. 14d.

Sometimes, however, both lengths and angles are important. For example, the various effects that can be caused by incorrect widths and incorrect angles are shown in Fig. 15, for a spanner gripping, or failing to grip, a square nut.



WIDTH AND ANGLES BOTH WRONG.

ANGLES RIGHT, WIDTH WRONG.

WIDTH RIGHT, ANGLES WRONG.

WIDTH AND ANGLES BOTH RIGHT.

CORRECT AND INCORRECT WIDTHS AND ANGLES.

FIG. 15.

By means of lengths and angles we can specify exactly any shape made up from straight lines. Some of these shapes are shown in Fig. 16.

/ 5.

5. AREAS.

5.1 We have seen that lengths can be measured in miles, yards, feet, inches, etc., and fractions or decimals of miles, yards, feet, inches, etc. A different kind of measure is needed to answer questions, such as -

> "How much ground does this concrete slab cover?", or
>
> "What ground surface must be marked out to dig this excavation?"

Both of these questions require a knowledge of areas.

5.2 Consider the shapes shown in Fig. 16.

Each of these shapes covers the same amount of paper, even though the shapes are different. This is expressed by saying that the areas of the shapes are equal. The names given to these shapes are -

(a) Square.
(b) Rectangle.
(c) Parallelogram.
(d) Triangle.

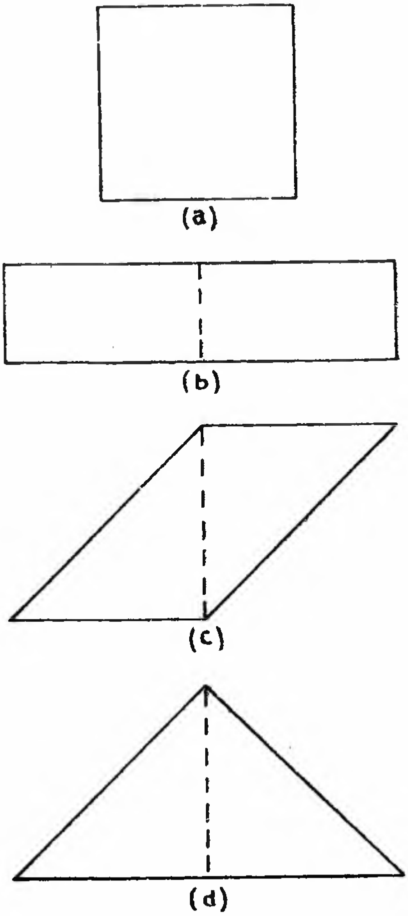
(There are many other shapes that could cover the same area, but those shown in Fig. 16 are the more common.)

We can always test whether two figures are equal in area by asking "Can I make this shape by cutting that one into pieces, and then fitting the pieces together again?" When this is possible the areas are equal.

From this we can see how areas are measured. We start with a standard piece, for example that shown in Fig. 16a which is known as a square. Each side is 1 inch long, and the area is, therefore, 1 square inch. This is often abbreviated to 1 sq. in.

Any other shape that can be made by cutting this square up and fitting the pieces together again also has an area of 1 square inch.

The shapes of Figs. 16b, 16c and 16d are made from the square shown in Fig. 16a, therefore, each has an area of 1 square inch. (In Figs. 16b, 16c and 16d, the dotted lines show where to cut these shapes, to get pieces which make the original square.)

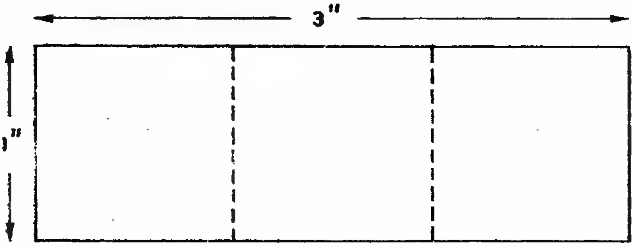


THE AREAS OF EACH SHAPE ARE EQUAL.

FIG. 16.

5.3 Sometimes, we can fill a space with a number of squares like Fig. 16a. For example, the shape of Fig. 17 which is 3 inches long and 1 inch high can be split up into three squares. So its area is 3 square inches.

Any shape which is made from Fig. 17 by cutting it up and moving the pieces around, also has an area of 3 square inches.



3 SQUARE INCHES.

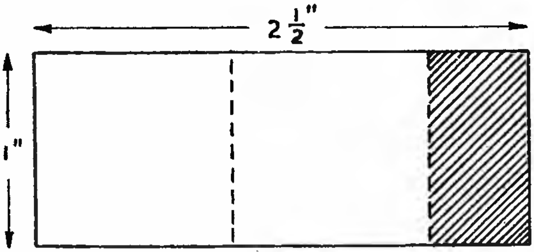
FIG. 17.

5.4 An example of a shape which introduces fractions is shown in Fig. 18.

This shape is 2-1/2 inches long and 1 inch high. Here we can put two squares in but the shaded part still remains.

How do we find the area of the shaded part?

We notice that two pieces, each exactly like the shaded part of Fig. 18 can be fitted together to make a single square (see Fig. 19a). Each must, therefore, be 1/2 square inch. So the shaded part of Fig. 18 is 1/2 square inch. The whole of Fig. 18, therefore, has an area of 2-1/2 square inches.



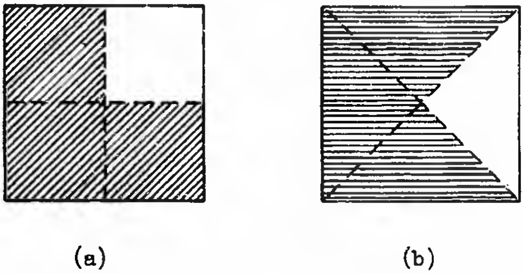
2-1/2 SQUARE INCHES.

FIG. 18.

THE SHADED PORTION HAS AN AREA OF 1/2 SQUARE INCH.

FIG. 19.

5.5 In the same way, we can show other fractions of a square inch. Figs. 20a and 20b show a square inch divided into four equal parts. Each part is, therefore, 1/4 of the whole, that is 1/4 square inch. So the shaded part is 3/4 square inch, and the unshaded part is 1/4 square inch.



THE SHADED PORTION HAS AN AREA OF 3/4 SQUARE INCH.

FIG. 20.

5.6 Units of Area. In the previous examples, the unit of measurement was taken as a square with 1 inch sides, giving an area of 1 square inch. In many cases, this unit is too small, and we select, for our unit, a square which has sides of 1 foot, 1 yard or even 1 mile. The areas of these squares are 1 square foot, 1 square yard and 1 square mile, respectively.

The following abbreviations are often used -

square foot ..... sq. ft.
square yard ..... sq. yd.
square mile ..... sq. mile.

Areas are also expressed in fractions of a sq. ft., sq. yd., sq. mile, etc.

5.7 Conversion of Units. It is often necessary to convert areas from one unit to another, for example, to convert square yards to square feet, square feet to square inches, etc.

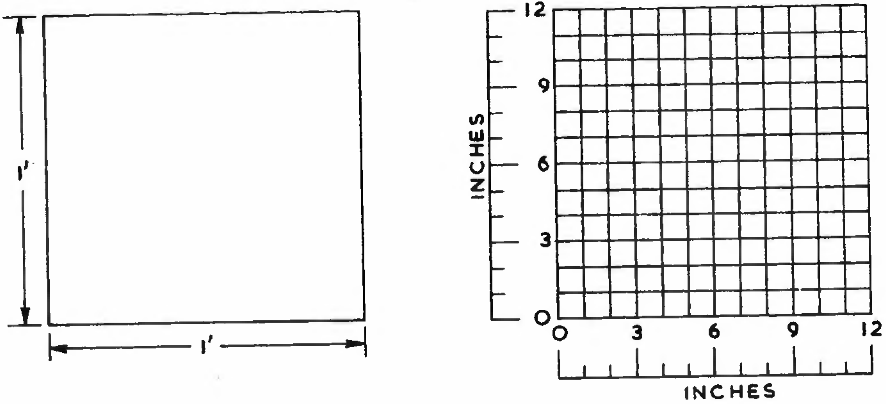
To find the number of square inches in a square foot we simply square the number of inches in a foot -

1 foot = 12 inches,

but 1 square foot does not equal 12 square inches.

1 square foot = 12^2 = 144 square inches.

This is shown in Fig. 21, which represents a square, the sides of which are each 1 foot long. Each side is marked off in inches and the opposite sides joined as shown in Fig. 21b. This process divides the large square into a number of small squares with 1 inch sides. The total number of 1 inch side squares contained in the whole square is 144 and, therefore, the area is 144 square inches.



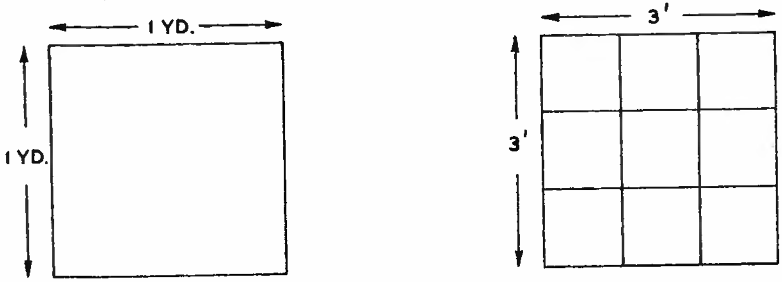
(a) (b)

1 SQUARE FOOT = 144 SQUARE INCHES.

FIG. 21.

By similar reasoning (see Fig. 22) -

1 square yard = 3^2 = 9 square feet.



(a) (b)

1 SQUARE YARD = 9 SQUARE FEET.

FIG. 22.

To convert square inches to square feet, divide by 144 -

36 square inches = $\frac{36}{144} = \frac{1}{4}$ square foot.

/ To

To convert square feet to square inches, multiply by 144 -

$$1\text{-}1/8 \text{ square feet } = \frac{9}{8} \times \frac{144}{1} = 162 \text{ square inches.}$$

To convert square feet to square yards, divide by 9 -

$$1\text{-}1/8 \text{ square feet } = 1\frac{1}{8} \div 9$$

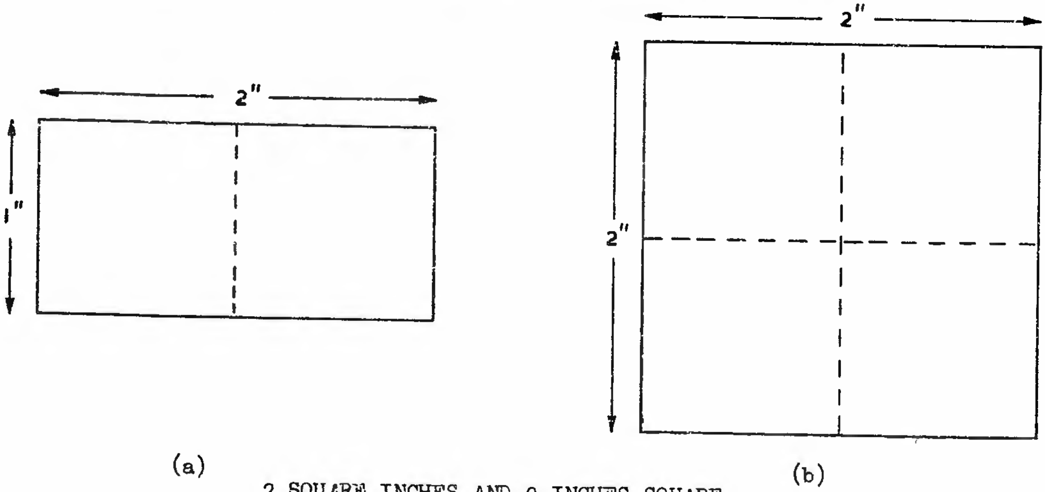
$$= \frac{9}{8} \times \frac{1}{9} = \frac{1}{8} \text{ square yard.}$$

To convert square yards to square feet, multiply by 9 -

$$1\text{-}1/2 \text{ square yards } = \frac{3}{2} \times \frac{9}{1} = 13\frac{1}{2} \text{ square feet.}$$

5.8 Square Feet and Feet Square. Confusion is often caused by the difference between two square inches and two inches square, three square feet and three feet square, etc.

Two square inches means an area which can be covered by two squares, each containing 1 square inch (see Fig. 23a). Two inches square means a square, each side of which is 2 inches long. (See Fig. 23b.) This square contains 4 square inches.

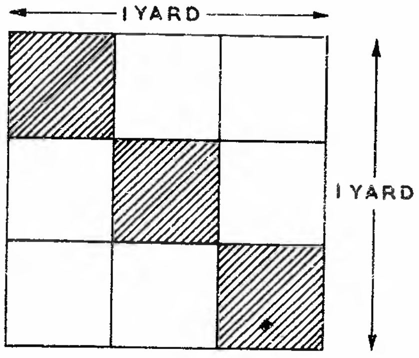


(a) (b)

2 SQUARE INCHES AND 2 INCHES SQUARE.

FIG. 23.

Similarly, 3 square feet and 3 feet square sound very similar, but they do not mean the same thing. The area in Fig. 24 represents 3 feet square, being 1 yard long and 1 yard wide. The shaded area represents an area of 3 square feet.



3 SQUARE FEET AND 3 FEET SQUARE.

FIG. 24.

6. CALCULATION OF AREAS.

6.1 Area of Square. A square is a surface enclosed by four equal straight lines, the four angles so formed being right angles.

In Fig. 25 the lines AB, BC, CD and DA are the four sides of a square each 6 inches long.

The larger square is divided into a number of small squares of 1 inch sides. The total number of small squares = 6 × 6 = 36, and the area of the large square is 36 square inches.

Therefore, to find the area of a square we square the length of one side.

When a square has the length expressed in feet, yards, miles, etc., the area is in square feet, square yards, square miles, etc.

Problem. What is the area of a square concrete slab with 4'6" sides.

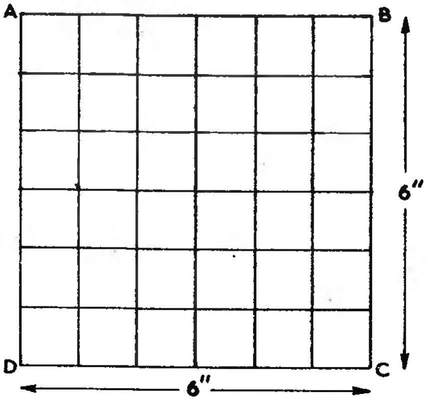
$$\text{Area} = (4'6")^2$$

$$= (4\tfrac{1}{2})^2$$

$$= \frac{9}{2} \times \frac{9}{2}$$

$$= \frac{81}{4}$$

$$= \underline{\underline{20\text{-}1/4 \text{ square feet}}}.$$



AREA OF A SQUARE.

FIG. 25.

6.2 Area of Rectangle. A rectangle is a surface enclosed by four straight lines. The opposite sides are equal and all four angles are right angles.

Fig. 26 shows a rectangle. The sides AB and CD are each equal to 6 inches, and the sides AD and BC are each equal to 3 inches. On dividing the rectangle into square inches as was done for the square, it is seen that there are three rows with 6 square inches in each. The area of the rectangle equals 6 × 3 = 18 square inches.

Therefore, to find the area of a rectangle, reduce the sides to the same units, inches, feet, yards or miles, etc., and then multiply the length of one side by the length of the adjacent side. This product is the area of the rectangle in square inches, square feet, square yards or square miles, etc.

Problem. What is the area of a concrete slab 5'4" by 4'6".

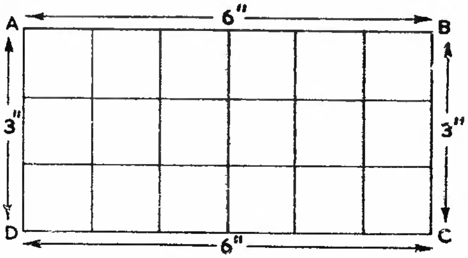
$$\text{Area} = 5'4" \times 4'6"$$

$$= 5\tfrac{1}{3} \times 4\tfrac{1}{2}$$

$$= \frac{\overset{8}{\cancel{16}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}}$$

$$= 8 \times 3$$

$$= \underline{\underline{24 \text{ square feet}}}.$$



AREA OF A RECTANGLE.

FIG. 26.

6.3 **Area of Right Angle Triangle.** A triangle is a surface enclosed by three straight sides. When one of the angles is a right angle, the triangle is termed a right angle triangle.

In Fig. 27, ABCD is a rectangle with diagonal BD drawn. The diagonal is the line drawn from one corner of the rectangle to the opposite corner. This diagonal divides the rectangle into two right angle triangles ABD and BCD. The two triangles are exactly similar, and are, therefore, equal in area.

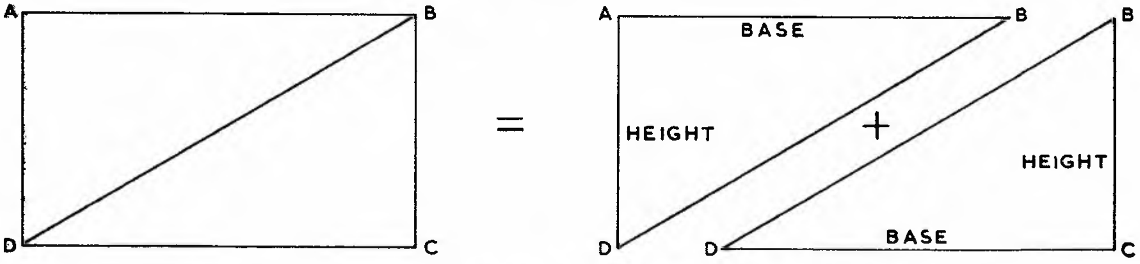


FIG. 27. A RECTANGLE EQUALS TWO RIGHT ANGLE TRIANGLES.

The area of the right angle triangle is thus half the area of the rectangle of which it forms part.

$$\text{Area of rectangle} = \text{length AB} \times \text{length AD, or}$$
$$= \text{length CD} \times \text{length CB.}$$
$$\text{Therefore, area of triangle} = \frac{1}{2}\,(\text{length AB} \times \text{length AD}), \text{ or}$$
$$= \frac{1}{2}\,(\text{length CD} \times \text{length CB}).$$

Expressed differently, the area of a right angled triangle equals half the base multiplied by the height, that is -

$$\text{Area} = \frac{1}{2}\,(\text{Base} \times \text{Height}).$$

Problem. What is the area of the right angle triangle shown in Fig. 28?

$$\text{Area} = \frac{1}{2}\,(\text{Base} \times \text{Height})$$
$$= \frac{1}{2} \times 6'9'' \times 2'8''$$
$$= \frac{1}{2} \times 6\frac{3}{4} \times 2\frac{2}{3}$$
$$= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{9}{\cancel{27}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}}$$
$$= \underline{\underline{\text{9 square feet}}}.$$

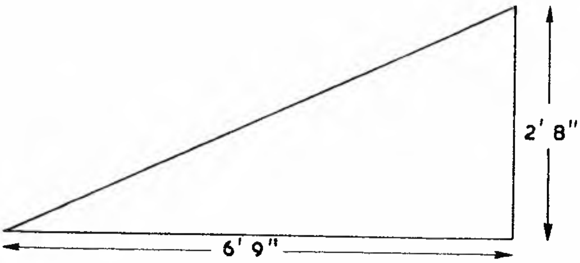


FIG. 28. RIGHT ANGLE TRIANGLE.

6.4 **Area of Parallelogram.** A parallelogram is a surface enclosed by four straight sides. The opposite sides are equal and parallel, as in a rectangle, but the four angles are not right angles. Fig. 29 shows a parallelogram ABCD.

The area of a parallelogram = base × height. In Fig. 29, the base is represented by the length of the sides AB or CD, and the height is the length of the dotted line AE which is drawn at right angles to CD.

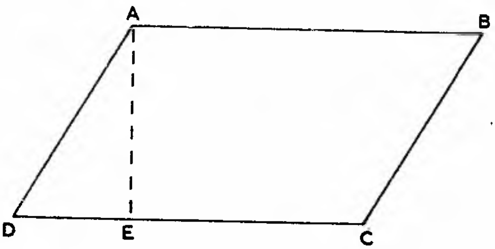


FIG. 29. PARALLELOGRAM.

Problem. What is the area of the parallelogram in Fig. 29 when the length of the side AB is 5'4" and the height AE is 2'3"?

$$\begin{aligned}\text{Area} &= \text{Base} \times \text{Height}\\ &= 5'4'' \times 2'3''\\ &= 5\tfrac{1}{3} \times 2\tfrac{1}{4}\\ &= \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}}\\ &= 4 \times 3\\ &= \underline{12 \text{ square feet}}.\end{aligned}$$

6.5 **Area of a Circle.** As the proof of the formulae for the area of a circle is beyond the scope of this course, the formulae given below should be memorised.

$$\begin{aligned}\text{Area of circle} &= \pi \times (\text{radius})^2\\ &= \pi r^2\\ &= \frac{22}{7} \times r^2.\end{aligned}$$

$$\begin{aligned}\text{Also, area of circle} &= \pi \times \left(\frac{\text{diameter}}{2}\right)^2\\ &= \frac{\pi d^2}{4}\\ &= \frac{11}{14} \times d^2.\end{aligned}$$

Problem. What is the area of a circular slab of concrete 1'9" radius?

$$\begin{aligned}\text{Area} &= \frac{22}{7} \times r^2\\ &= \frac{22}{7} \times \left(\frac{7}{4}\right)^2\\ &= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{4}}} \times \frac{7}{4}\\ &= \frac{11 \times 7}{2 \times 4}\\ &= \frac{77}{8}\\ &= \underline{9\text{-}5/8 \text{ square feet}}.\end{aligned}$$

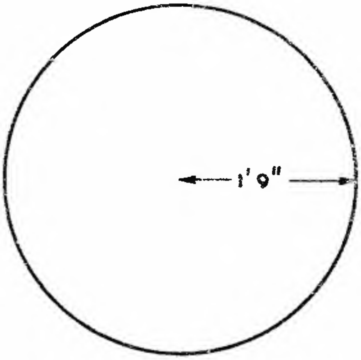


FIG. 30. AREA OF CIRCLE.

The area could also be calculated from the diameter, which is (1'9" × 2) = 3'6".

$$\begin{aligned}\text{Area} &= \frac{11}{14} \times d^2\\ &= \frac{11}{14} \times \left(\frac{7}{2}\right)^2\\ &= \frac{11}{\underset{2}{\cancel{14}}} \times \frac{\overset{1}{\cancel{7}}}{2} \times \frac{7}{2}\\ &= \frac{11 \times 7}{2 \times 2 \times 2}\\ &= \frac{77}{8}\\ &= \underline{9\text{-}5/8 \text{ square feet}}.\end{aligned}$$

/ 6.6

6.6 Area of a Circular Ring. Fig. 31 shows a ring made by cutting a circle of radius r from one of radius R.

The area of the larger circle = πR^2.

The area of the smaller circle = πr^2.

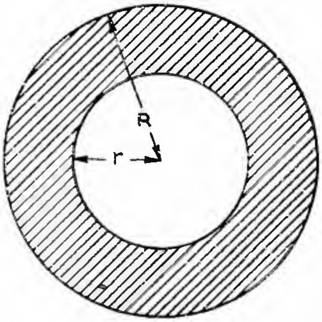
$$\text{The area of the ring} = \pi R^2 - \pi r^2$$

$$= \pi \left[R^2 - r^2\right].$$

$$\text{Also, the area of the ring} = \pi \left[\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$= \pi \left[\frac{D^2}{4} - \frac{d^2}{4}\right]$$

$$= \frac{\pi}{4}\left[D^2 - d^2\right].$$



CIRCULAR RING.

FIG. 31.

Problem. A pole of diameter 12" is set in concrete, which extends for a distance of 9" all around the pole. (See Fig. 32.) What is the surface area of the concrete?

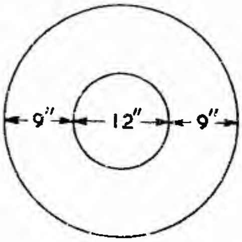
Smaller diameter (d) = 12" = 1'

Larger diameter (D) = 9" + 12" + 9"

= 30"

= 2'6"

$= \frac{5}{2}$ feet.



AREA OF CIRCULAR RING.

FIG. 32.

$$\text{Area of concrete} = \frac{\pi}{4}\left[D^2 - d^2\right]$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left[\left(\frac{5}{2}\right)^2 - (1)^2\right]$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left[\left(\frac{5}{2}\right)\times\left(\frac{5}{2}\right) - (1 \times 1)\right]$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left[\frac{25}{4} - 1\right]$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left[\frac{25}{4} - \frac{4}{4}\right]$$

$$= \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \frac{1}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{21}}}{4}$$

$$= \frac{11 \times 3}{2 \times 4}$$

$$= \frac{33}{8}$$

= 4-1/8 square feet.

/ 7.

7. VOLUMES.

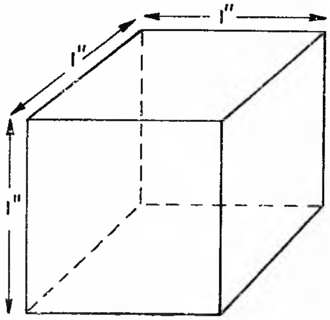
7.1 In Section 5, dealing with the calculation of areas, the surfaces had two dimensions, length and breadth.

A solid has three dimensions, length, breadth and depth (or height). The volume of a solid is measured by the number of unit volumes which are contained in the solid in the same way as the area of a surface is measured by the number of unit areas which it contains.

Solids are divided into many groups but only those commonly met in lines practice are covered in these notes.

7.2 Cube. A cube (see Fig. 33) has its sides and ends all equal squares. There are then six faces to a cube, each surface being a square of the same size.

7.3 Units of Volume. The unit of volume is a cube with sides of equal length. For example, a cubic inch is the volume of a cube with all sides equal to one inch.



1" CUBE.

FIG. 33.

Volumes can be measured in cubic feet or cubic yards, instead of in cubic inches. A cube one foot in length, breadth and height has a volume of one cubic foot. A cube one yard in length, breadth and height has a volume of one cubic yard. Many materials, for example, sand, gravel, etc., are usually sold by the cubic yard. The following abbreviations are often used -

cubic inch .... cub. in.
cubic foot .... cub. ft.
cubic yard .... cub. yd.

7.4 Conversion of Units. It is often necessary to convert volumes from one unit to another, for example, to convert cubic yards to cubic feet, cubic feet to cubic inches, etc.

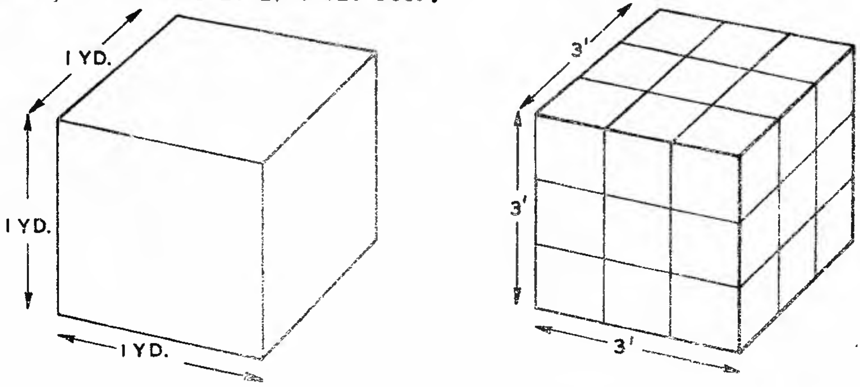
To find the number of cubic feet in a cubic yard -

1 yard = 3 feet,

but 1 cubic yard does not equal 3 cubic feet.

1 cubic yard = 3 × 3 × 3 = 27 cubic feet.

This is shown in Fig. 34 which represents a cube, the sides of which are each 1 yard long. Each side is marked off in feet and the opposite sides joined as shown in Fig. 34b. This process divides the large cube into a number of smaller cubes with 1 foot sides. The total number of smaller cubes contained in the larger cube is 27 and, therefore, the volume is 27 cubic feet.



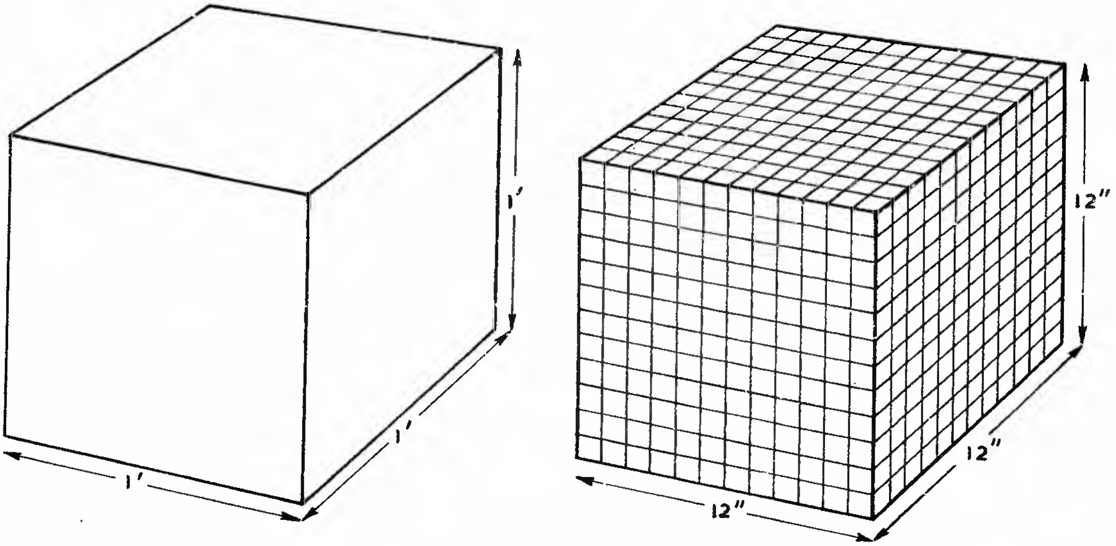
(a) (b)

1 CUBIC YARD = 27 CUBIC FEET.

FIG. 34.

By similar reasoning, (see Fig. 35) -

$$1 \text{ cubic foot} = 12 \times 12 \times 12 = 1728 \text{ cubic inches.}$$



1 CUBIC FOOT = 1728 CUBIC INCHES.

FIG. 35.

To convert cubic inches to cubic feet, divide by 1728 -

$$648 \text{ cubic inches} = \frac{648}{1728} = \frac{3}{8} \text{ cubic foot.}$$

$$691 \text{ cubic inches} = \frac{691}{1728} = 0.4 \text{ cubic foot.}$$

To convert cubic feet to cubic inches, multiply by 1728 -

$$1/8 \text{ cubic foot} = \frac{1}{8} \times \frac{1728}{1} = 216 \text{ cubic inches.}$$

To convert cubic feet to cubic yards, divide by 27 -

$$6\text{-}3/4 \text{ cubic feet} = 6\frac{3}{4} \div 27$$

$$= \frac{27}{4} \times \frac{1}{27} = \frac{1}{4} \text{ cubic yard.}$$

To convert cubic yards to cubic feet, multiply by 27 -

$$1\text{-}1/3 \text{ cubic yards} = \frac{4}{3} \times \frac{27}{1} = 36 \text{ cubic feet.}$$

7.5 Liquid Measurement of Volumes. Volumes are also measured in pints and gallons. These units are used for liquids.

A table of liquid measurement is given in the Section of Tables at the beginning of this book.

/ 8.

8. CALCULATION OF VOLUMES.

8.1 Cube. A cube with 3 inch sides is shown in Fig. 36. The smaller cubes represent cubic inches. To find the volume, multiply the area of one end by the height of the cube.

Volume = area of end × height

= length × breadth × height

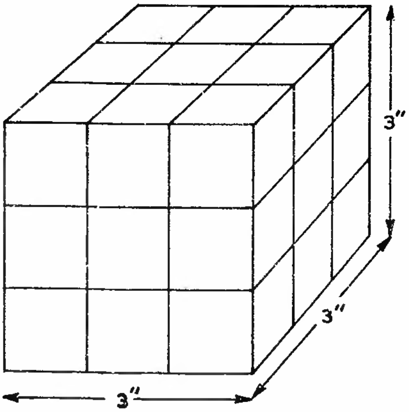
= 3" × 3" × 3"

= 27 cubic inches.

Or volume = $\frac{1'}{4} \times \frac{1'}{4} \times \frac{1'}{4}$

= 1/64 cubic foot.

(Therefore, 64 of these cubes would fill a volume of 1 cubic foot.)



VOLUME OF A CUBE.

FIG. 36.

8.2 Rectangular Prism. The cube shown in Fig. 36 is also known as a rectangular prism. The sides are at right angles to the ends. Another example of a rectangular solid is shown in Fig. 37. This figure is not a cube because the lengths of the sides are unequal.

Volume = length × breadth × height

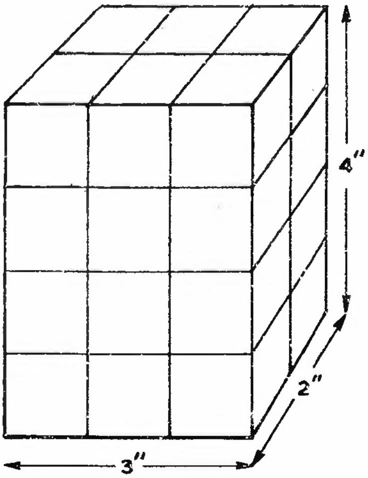
= 3" × 2" × 4"

= 24 cubic inches.

Or volume = $\frac{1'}{4} \times \frac{1'}{6} \times \frac{1'}{3}$

= 1/72 cubic foot.

(Therefore, 72 of these solids would fill a volume of 1 cubic foot.)



VOLUME OF A RECTANGULAR SOLID.
(NOT A CUBE.)

FIG. 37.

Problem. A trench of the following dimensions has to be excavated -

Length - 50 yards.
Width - 15 inches.
Depth - 2 feet.

What is the volume of the excavation?

$$\text{Volume} = \text{length} \times \text{width} \times \text{depth}.$$

(Remember to keep all the dimensions in similar units, for example, inches, feet or yards. In this case, we select feet.)

$$\text{Volume} = (50 \times 3) \times \frac{15}{12} \times 2$$

$$= \frac{50}{1} \times \frac{3}{1} \times \frac{15}{12} \times \frac{2}{1}$$

$$= \frac{\overset{25}{\cancel{50}}}{1} \times \frac{\overset{1}{\cancel{3}}}{1} \times \frac{15}{\underset{\underset{\underset{1}{\cancel{2}}}{\cancel{6}}}{\cancel{12}}} \times \frac{\overset{1}{\cancel{2}}}{1}$$

$$= 25 \times 15$$

$$= \underline{\underline{375 \text{ cubic feet}}}.$$

$$\begin{array}{r} 25 \\ \underline{15} \\ 125 \\ \underline{250} \\ 375 \end{array}$$

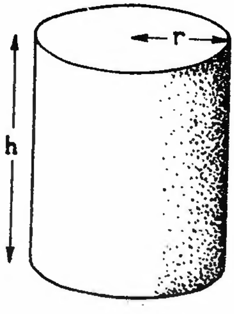
8.3 **Cylinder.** A cylinder is a special type of prism in which the ends are circles, parallel to one another and equal in area.

The volume of the cylinder (see Fig. 38) equals the area of one end multiplied by the height.

$$\text{Area of end} = \pi \times (\text{radius})^2.$$

$$\text{Therefore, volume} = \pi \times (\text{radius})^2 \times \text{height}$$

$$= \pi r^2 h \text{ or } \frac{\pi d^2 h}{4}$$



VOLUME OF CYLINDER.

FIG. 38.

Problem. A cylindrical kerosene drum is 11" in diameter and 12" high. What volume of liquid does it contain when full?

$$\text{Volume} = \frac{\pi d^2 h}{4}$$

$$= \frac{22}{7} \times \frac{1}{4} \times \left(\frac{11}{1}\right)^2 \times \frac{12}{1}$$

$$= \frac{22}{7} \times \frac{1}{\underset{1}{\cancel{4}}} \times \frac{11}{1} \times \frac{11}{1} \times \frac{\overset{3}{\cancel{12}}}{1}$$

$$= \frac{22}{7} \times \frac{363}{1}$$

$$= \frac{7986}{7}$$

$$= \underline{\underline{1141 \text{ cubic inches}}}. \quad \text{(Approximately.)}$$

$$\begin{array}{r} 363 \\ \underline{22} \\ 726 \\ \underline{7260} \\ 7)\underline{7986} \\ 1140\text{-}6/7. \end{array}$$

/ 9.

9. SURFACE AREAS OF VOLUMES.

9.1 Cube. A cube has 6 sides all of equal area.

$$\text{Total surface area} = 6 \times \text{area of one side}$$

$$= 6 \times (\text{length of one side})^2.$$

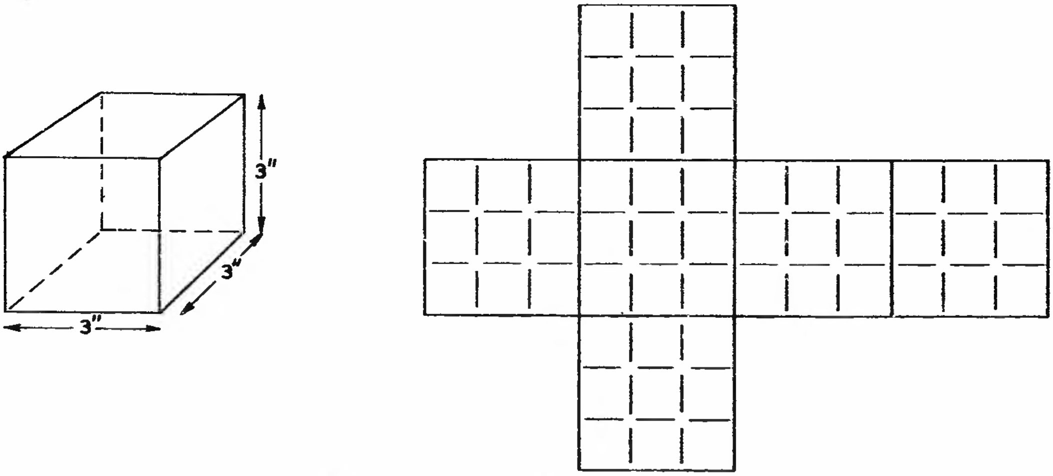


FIG. 39. SURFACE AREA OF CUBE.

The surface area of the cube shown in Fig. 39 -

$$= 6 \times 3^2$$

$$= 6 \times 3 \times 3$$

$$= \underline{54 \text{ square inches}}.$$

9.2 Rectangular Prism. The rectangular prism shown in Fig. 40 has three pairs of sides of unequal areas.

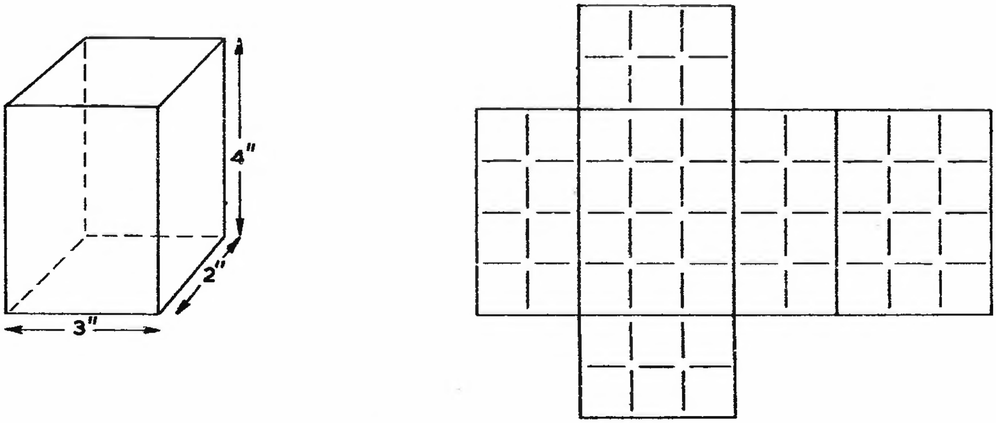


FIG. 40. SURFACE AREA OF RECTANGULAR PRISM.

The total surface area -

$$= 2(\text{length} \times \text{breadth}) + 2(\text{length} \times \text{height}) + 2(\text{breadth} \times \text{height}).$$

$$= 2\left[(L \times B) + (L \times H) + (B \times H)\right].$$

/ The

The surface area of the rectangular prism shown in Fig. 40 -

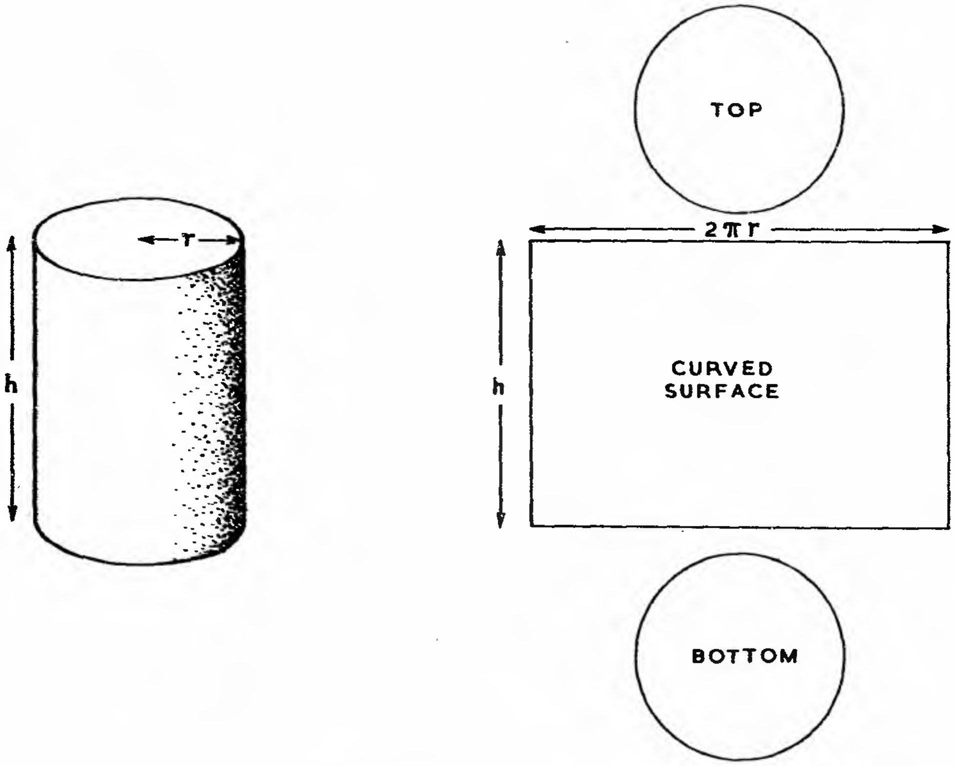
$$= 2\left[(3 \times 2) + (3 \times 4) + (2 \times 4)\right]$$

$$= 2\left[6 + 12 + 8\right]$$

$$= 2 \times 26$$

= 52 square inches.

9.3 Cylinder. Suppose a piece of paper is wrapped tightly around the curved surface of the cylinder shown in Fig. 41, in such a way as just to cover the curved surface without overlap. When this paper is then opened out flat, the area of the paper equals the area of the curved surface of the cylinder.



SURFACE AREA OF CYLINDER.

FIG. 41.

The area of the curved surface of a cylinder, therefore -

= the distance around the edge at the upper or lower end × height

$$= 2\pi rh.$$

The total surface area -

= the area of the curved surface + the areas of the two ends

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r\ (h + r).$$

/ Problem.

Problem. Find the total surface area of a cylinder, the diameter of the ends being 14" and the height 12".

$$\text{Radius} = \frac{\text{Diameter}}{2}$$

$$= \frac{14}{2}$$

$$= 7 \text{ inches.}$$

$$\text{Total area} = 2\pi r(h + r)$$

$$= \frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} \times (12 + 7)$$

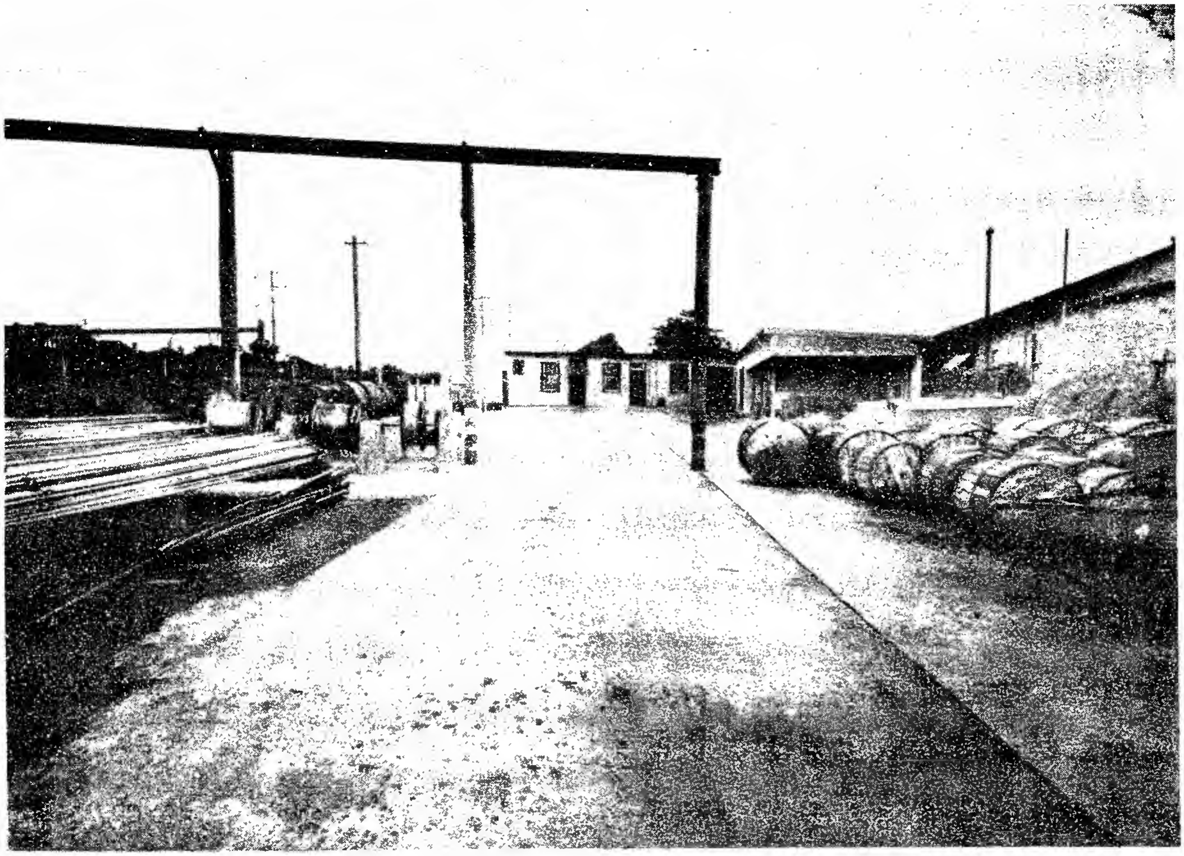
$$= \frac{2}{1} \times \frac{22}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{1} \times \frac{19}{1}$$

$$= 2 \times 22 \times 19$$

$$= 44 \times 19$$

$$= \underline{836 \text{ square inches.}}$$

$$\begin{array}{r} 44 \\ \underline{19} \\ 396 \\ \underline{440} \\ 836 \end{array}$$



TYPICAL LINE DEPOT.

10. SAWN TIMBER MEASUREMENTS.

10.1 Timber sawn into boards, battens, etc., is sold at so much per 100 superficial feet. The standard thickness of such timber is 1 inch.

A board 1 foot long, 1 foot wide, and 1 inch thick is taken to be one superficial foot. The solid contents of such a board is only one-twelfth of a cubic foot (that is, 1 foot × 1 foot × 1/12 foot).

When, therefore, the volume of a certain quantity of sawn timber is 15 cubic feet, then, according to the timber trade measurements, it contains 180 superficial feet (that is, 15 × 12).

The term "superficial feet" is often abbreviated to "super. ft."

Problem No. 1. When 6 boards, each 8 feet long, 6 inches wide, and 1 inch thick, are purchased at a timber yard, how many superficial feet do the boards contain?

$$\text{Volume} = 6 \times 8 \text{ ft.} \times \frac{1}{2} \text{ ft.} \times \frac{1}{12} \text{ ft.}$$

$$= 2 \text{ cubic feet.}$$

To convert to superficial feet, multiply the volume in cubic feet by 12.

$$= 2 \times 12 \text{ superficial feet}$$

$$= \underline{\underline{24 \text{ superficial feet}}}.$$

Problem No. 2. Find the number of superficial feet in 20 boards, each 9 feet long, 8 inches wide, and 1-1/2 inches thick.

$$\text{Volume} = 20 \times 9 \text{ ft.} \times \frac{2}{3} \text{ ft.} \times \frac{1}{8} \text{ ft.}$$

$$= 15 \text{ cubic feet}$$

$$= 15 \times 12 \text{ superficial feet}$$

$$= \underline{\underline{180 \text{ superficial feet}}}.$$

Problem No. 3. Find the number of superficial feet in 10 pieces of hardwood, each 12 feet long, 4 inches wide, and 3 inches thick.

$$\text{Volume} = 10 \times 12 \text{ ft.} \times \frac{1}{3} \text{ ft.} \times \frac{1}{4} \text{ ft.}$$

$$= 10 \text{ cubic feet}$$

$$= 10 \times 12 \text{ superficial feet}$$

$$= \underline{\underline{120 \text{ superficial feet}}}.$$

Problem No. 4. How many superficial feet of timber can be obtained from a cylindrical log 12 feet long and 11 feet 4 inches in girth (circumference)?

A cylindrical log, after allowing for waste, will cut up to a piece of timber with square ends, the side of which are equal to the quarter-girth of the log when in the round. In this case the quarter-girth is 2 feet 10 inches (that is 1/4 of 11 feet 4 inches). Now, if the boards are to be 1 inch thick, there will be 34 boards each 1 inch thick, 12 feet long, and 2 feet 10 inches wide -

$$= 34 \times 12 \times 2\frac{5}{6}$$

$$= \underline{\underline{1,156 \text{ superficial feet}}}.$$

10.2 Floorings, linings and weatherboards are usually sold by linear or running feet; for example, 12 feet of flooring board 6 inches in width and 1 inch thick = 12 feet of flooring board.

/ 11.

11. TEST QUESTIONS.

1. How many degrees in a right angle?

2. What is meant by the terms -

   (i) included angle, and

   (ii) angle of deviation,

   applied to aerial line construction? Are these angles greater or less than a right angle?

3. What is the ratio between the circumference and radius of a circle?

4. A length of cable is wound in a coil approximately 1'9" in diameter. There are 18 turns in the coil. What is the approximate length of the cable in yards?

5. How many square yards in 4,050 square inches.

6. What is the volume in cubic yards of a concrete path 12 yards long, 2 feet 3 inches wide, and 4 inches deep?

7. Five manholes each 5' × 4' × 5' are to be excavated and the excavated soil removed. A truck with a maximum capacity of 5 cubic yards is available. How many trips are necessary for this truck to remove the soil?

8. The dimensions of a kerosene tin are 9-1/4" × 9-1/4" × 13". How many gallons will this tin hold? (1 gallon = 277-1/4 cubic inches, approximately).

9. It is required to excavate a hole having the following dimensions -

   2-1/2 yards wide, 3 yards long and 3 yards deep.

   Assuming a man can excavate 10 cubic feet in 35 minutes, how long will he take to excavate the hole? (Give answer to the nearest 1/4 hour.)

10. A trench of the following dimensions has to be excavated -

    Length - 40 yards,
    Width - 15 inches,
    Depth - 18 inches.

    If a man can excavate 1 cubic yard in 2-1/2 hours, how long will three men take to dig the trench. (Give answer to the nearest 1/4 hour.)

END OF PAPER.

COMMONWEALTH OF AUSTRALIA.

Engineering Branch,
Postmaster-General's Department,
Treasury Gardens,
Melbourne. C.2.

BASIC MATHEMATICS FOR LINEMEN-IN-TRAINING.

RIGHT ANGLE TRIANGLES.

CONTENTS:

---

1. INTRODUCTION.

1.1 A knowledge of right angle triangles is essential to the solution of many problems which arise in normal lines construction. This knowledge enables us to solve quickly and accurately such problems as -

(i) the line of stay wires on an aerial line route,

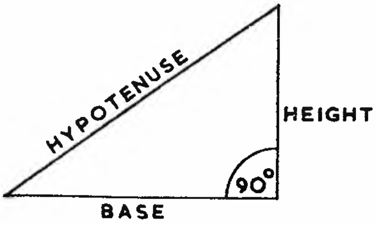
(ii) the distance across a river,

(iii) the height of a pole or tree,

(iv) the length of a stay wire,

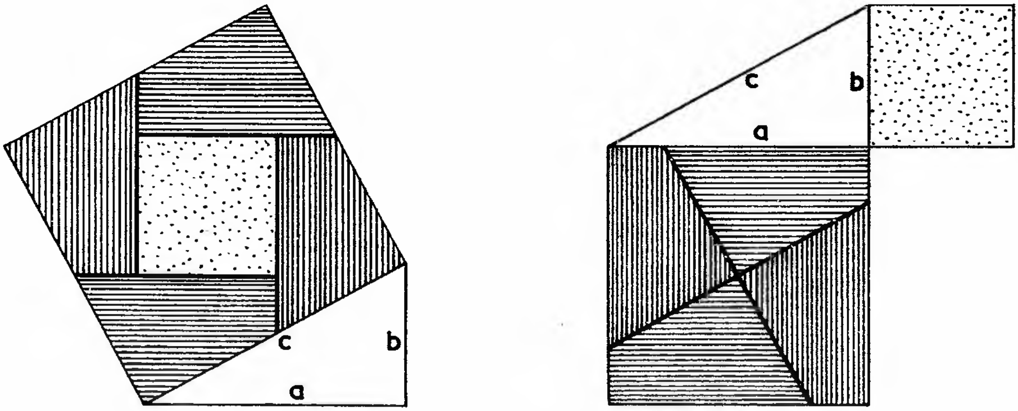
and many other similar problems.

1.2 A right angle triangle (see Fig. 1) is a figure with three sides, in which one of the angles equals 90°, that is, a right angle. The sum of the other two angles equals 90°, but each of them is less than a right angle. The two sides which enclose the right angle are called the base and height respectively. The side opposite the right angle is called the hypotenuse.

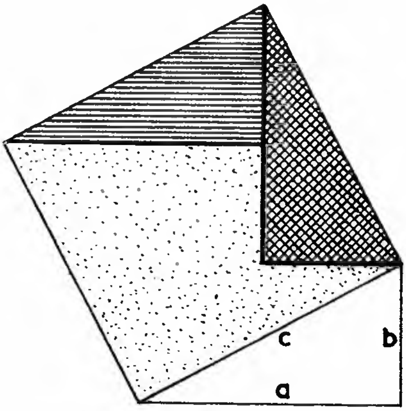


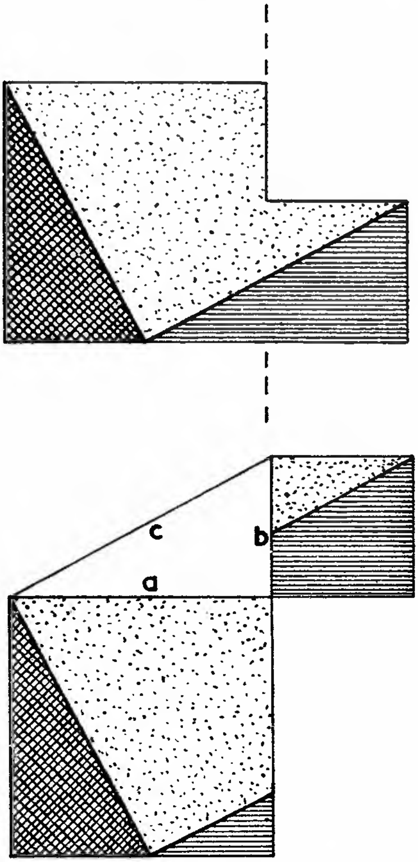
RIGHT ANGLE TRIANGLE.

FIG. 1.



Pythagoras' theorem states that the square on the hypotenuse (c), contains just enough material to make the square on the base (a), and the square on the height (b). One way of cutting up the square on c and making the other two squares is shown above. Another method is explained below.





The square on c is first divided into three parts. The dotted piece stays still. The other two pieces are moved. Then a straight cut along the dotted line gives the square on a and the square on b.

FIG. 2. PROOF OF PYTHAGORAS' THEOREM.

2. PYTHAGORAS' THEOREM.

2.1 There is a definite relationship between the three sides of a right angle triangle. This relationship is summarised in the following statement, which is known as Pythagoras' theorem -

> "The square on the longest side of a right angle triangle (that is, the hypotenuse) equals the sum of the squares on the other two sides."

Expressed as a formula -

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2.$$

The illustrations on Page 2 show two simple proofs of this theorem.

2.2 Let us now apply this theorem to finding the length of the hypotenuse of a right angle triangle when the length of the other two sides are known.

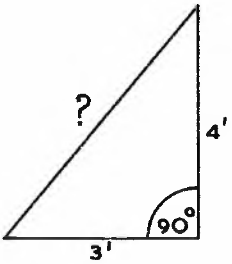
Problem. Calculate the length of the hypotenuse of the right angle triangle shown in Fig. 3, when the lengths of the other two sides are 3' and 4' respectively.

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2.$$

To find the length of the Hypotenuse, take the square root of both sides of the equation (see paragraph 5.1 of Paper No. 3).

$$\sqrt{\text{Hypotenuse}^2} = \sqrt{\text{Base}^2 + \text{Height}^2}$$

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{\text{Base}^2 + \text{Height}^2}\\ &= \sqrt{3^2 + 4^2}\\ &= \sqrt{9 + 16}\\ &= \sqrt{25}\\ &= \underline{\underline{5 \text{ feet}}}.\end{aligned}$$



WHAT IS LENGTH OF HYPOTENUSE?

FIG. 3.

2.3 Fig. 3 is, therefore, a right angle triangle in which the lengths of the three sides are 3', 4' and 5' respectively. When the lengths of the two sides of the right angle triangle are increased to 6' and 8', the length of the hypotenuse is 10'. When increased to 9' and 12', the hypotenuse is 15'.

From this problem, it is important to note that when the lengths of the three sides of a triangle have a ratio 3 : 4 : 5, the triangle is always a right angle triangle.

There is no very obvious reason why a ratio 3: 4 : 5 gives a right angle triangle. It is not because 3, 4 and 5 are numbers that follow each other. We can see this by trying other sets of numbers that follow each other, like 2, 3, 4 or 9, 10, 11. They do not give right angle triangles. On the other hand, 6, 8, 10 and 5, 12, 13 are not sets of numbers that follow immediately after each other, but they do give right angle triangles.

2.4 From the above discussion we can make these observations -

(i) When the square on the longest side of a triangle equals the sum of the squares on the other two sides, the triangle is a right angle triangle. The right angle is always opposite the longest side.

(ii) A right angle triangle can be drawn by constructing a triangle in which the three sides have a ratio 3 : 4 : 5.

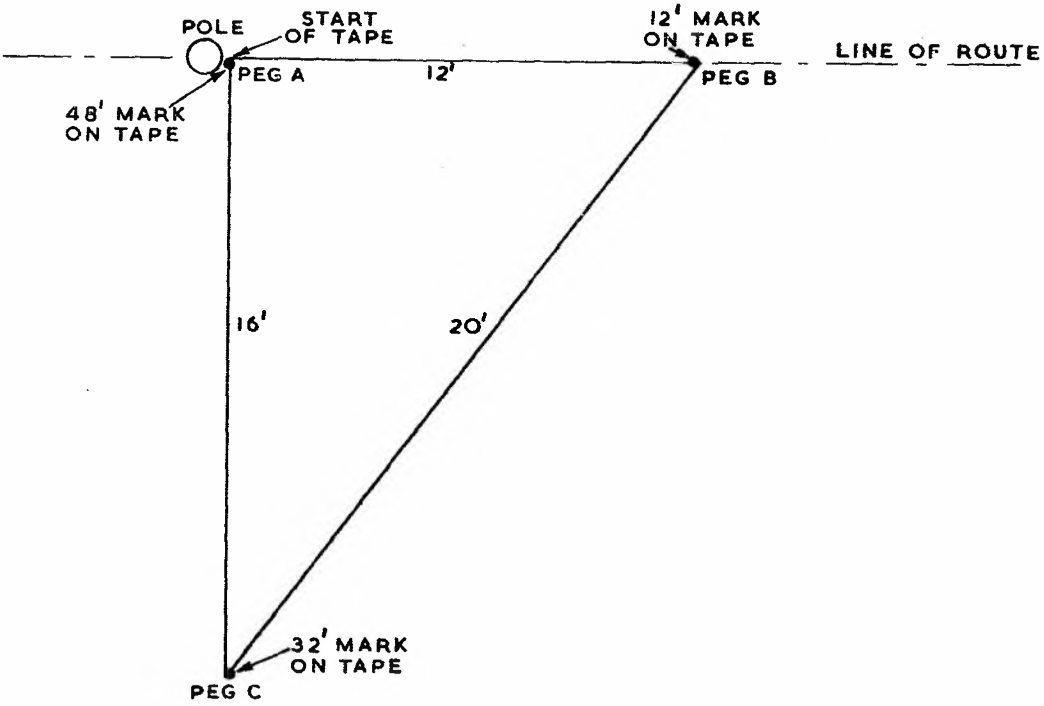
/3.

3. SETTING OUT A RIGHT ANGLE.

3.1 It is often necessary to set out a line at right angles to another line, for example, in the case of stays which are placed at right angles to a pole route, or during the survey of a route (see Section 5). The line at right angles can be set out by the 3, 4, and 5 method.

3.2 As explained in Section 2, when a triangle is made so that its sides are 3, 4 and 5 feet, or any multiple of these figures, it contains a right angle. Dimensions of 12', 16' and 20' are commonly used and have been selected for this explanation.

3.3 To locate a line at right angles to the line of route, a right angle triangle is first made with a measuring tape or a suitable cord marked at the required distances, as shown in Fig. 4.



SETTING OUT A RIGHT ANGLE.

FIG. 4.

Peg A is placed in the ground at the base of the pole and peg B at a point 12' from the pole and in direct line with the route. The start and the 48' mark of a tape are held firmly against peg A. The looped tape is then passed around peg B, which should show the 12' mark at that point. Peg C is held at the 32' mark on the tape and, when the tape is adjusted so that the tension between all pegs is equal, the peg is driven into the ground at this point. A cord held taut between pegs A and C gives a line at right angles to the route.

4. FINDING LINE OF STAY.

4.1 When a stay is to be fitted at right angles to the route, its position is found by the 3, 4, 5 method as shown in Section 3, or by the following method -

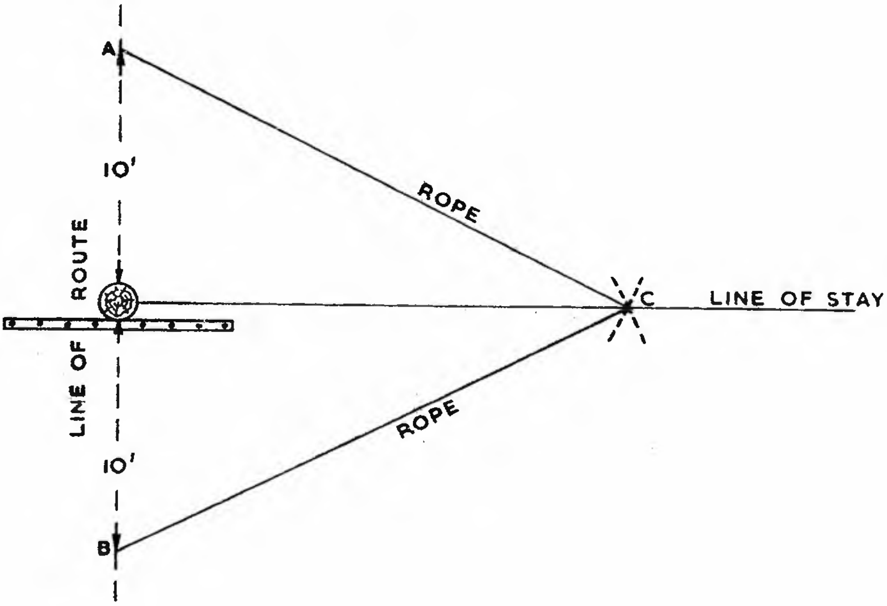


FIG. 5. STAY AT RIGHT ANGLES TO ROUTE.

Two pegs A and B are placed in direct line with the route 10' on either side of the pole as shown in Fig. 5. Two ropes of equal length are attached to the pegs and stretched out in the approximate direction of the stay. A peg C is placed at the point where the ropes meet. The line between peg C and the pole shows the line of the stay.

4.2 This method is also used to find the line of the stay at an angle pole. (See Fig. 6.)

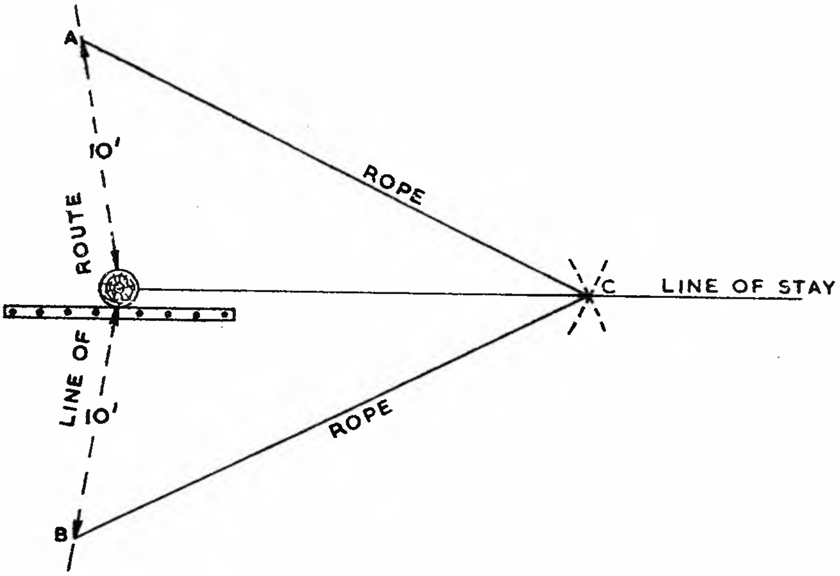


FIG. 6. STAY AT ANGLE POLE.

5. FINDING DISTANCE ACROSS A RIVER.

5.1 During the survey of a route it may be necessary to find the distance across a river, gully or some other obstruction. A method of measuring this distance is shown in Fig. 7.

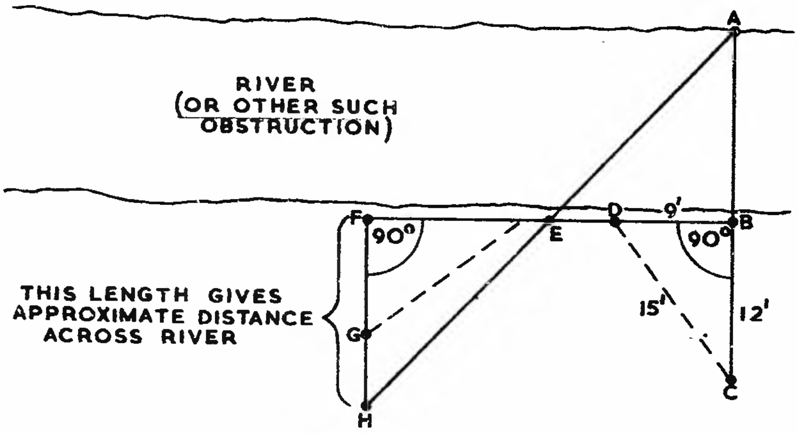


FIG. 7. MEASURING DISTANCE ACROSS A RIVER.

Select a prominent object A on the opposite bank of the river. Drive a peg B in the bank as nearly square across the river as possible. Drive peg C into the ground so that points A, B and C are in a straight line. Form a right angle at B using the 3', 4', 5' method or any multiple, say 9', 12', 15' as shown. The line BD is now at right angles to the line AB. From B, measure any convenient distance through D to point F. Place a stake at E midway between points B and F. At point F, set off a line FG at right angles to the line BF in the same manner as was done at point B. Extend this line to a point at which peg E comes into direct line with the object A. Call this point H. The distance from peg F to point H equals the distance from peg B to point A, and so gives the approximate distance across the river.

6. FINDING HEIGHT OF A POLE OR TREE.

6.1 The approximate height of a pole or tree can be found by the "shadow method."

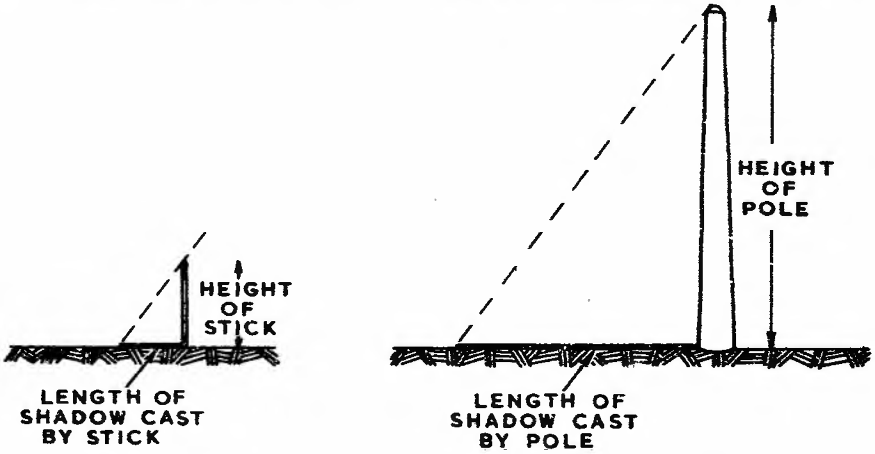


FIG. 8. FINDING HEIGHT OF POLE BY SHADOW METHOD.

Drive a stick into the ground in a vertical position. Measure the height from the ground to the top of the stick. A convenient height is about 6'. Measure the length of the shadow cast by the stick due to the sun. Then measure the length of the shadow cast by the pole or tree.

The height of the pole is found by direct proportion (see paragraph 5.4 of Paper No. 3).

$$\frac{\text{Height of Pole}}{\text{Length of Pole Shadow}} = \frac{\text{Height of Stick}}{\text{Length of Stick Shadow}}.$$

(Multiply each side of the equation by "Length of Pole Shadow".)

$$\text{Height of Pole} = \frac{\text{Height of Stick}}{\text{Length of Stick Shadow}} \times \frac{\text{Length of Pole Shadow}}{1}$$

Problem. The length of the shadow cast by a stick which is 6' out of the ground is 4'3". What is the height of a pole which casts a shadow 17' in length?

$$\text{Pole Height} = \frac{\text{Stick Height}}{\text{Stick Shadow}} \times \frac{\text{Pole Shadow}}{1}$$

$$= \frac{6}{4\frac{1}{4}} \times \frac{17}{1}$$

$$= \frac{6}{1} \times \frac{4}{\underset{1}{\cancel{17}}} \times \frac{\overset{1}{\cancel{17}}}{1}$$

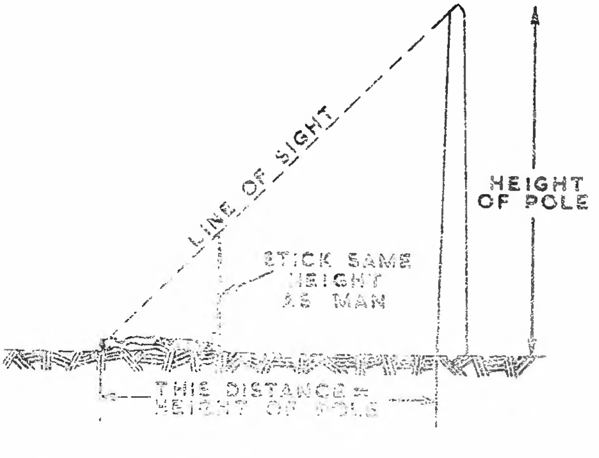
$$= 6 \times 4$$

$$= \underline{\underline{24 \text{ feet}}}.$$

6.2 When no shadow is cast from the sun, the method in Fig. 9 can be used. A stick is set vertically in the ground some distance from the pole. The height of the stick out of the ground is the same as the height of the man conducting the measurement.

The observer lies horizontally on the ground with his feet touching the stick, and sights the top of the pole with the top of the stick. If these are not in line, the procedure is repeated until they are.

The distance along the ground from the head of the observer to the base of the pole equals the height of the pole.



FINDING HEIGHT OF POLE BY SIGHTING METHOD.

7. FINDING LENGTH OF STAY.

7.1 A practical use of Pythagoras' theorem is finding the length of a stay. The method of doing this is shown in the following problems -

Problem. A stay is attached to a pole 20' from the ground. The distance from the pole to the point where the stay emerges from the ground is 22'. What is the approximate length of the stay wire?

Fig. 10 shows the conditions, and this figure represents a right angle triangle.

Let L = length of stay (in feet),

H = height of pole from ground level to the point of attachment of the stay (in feet),

D = distance from pole to point where stay emerges from the ground (in feet).

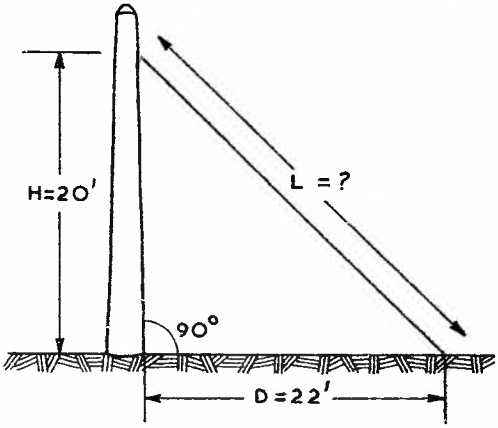
Then $L^2 = H^2 + D^2$.

Taking the square root of both sides -

$$\sqrt{L^2} = \sqrt{H^2 + D^2},$$

but the square root of L^2 is L,

therefore, $L = \sqrt{H^2 + D^2}$.



WHAT IS LENGTH OF STAY?

FIG. 10.

Substituting the values for H and D in this formula -

$$L = \sqrt{(20)^2 + (22)^2}$$

$$= \sqrt{400 + 484}$$

$$= \sqrt{884}$$

$$\begin{array}{r|l}
 & 2\ \ 9\ .\ 7\ \ 3 \\
\hline
2 & 8'84'.00'00 \\
 & 4 \\
49 & 484 \\
 & 441 \\
587 & \ \ 4300 \\
 & \ \ 4109 \\
5943 & \ \ \ \ 19100 \\
 & \ \ \ \ 17829 \\
 & \ \ \ \ \ \ 1271
\end{array}$$

Therefore, L = 29.73 feet

= 29'9" (approximately).

Length of stay is about 29'9".

/7.2

7.2 When the dimensions H and D are equal, the dimension L is found by the formula -

$$L = 1.414 \times H.$$

The proof of this is as follows -

$$L = \sqrt{H^2 + D^2}$$

$$\text{but } H^2 = D^2$$

$$\text{therefore, } L = \sqrt{H^2 + H^2}$$

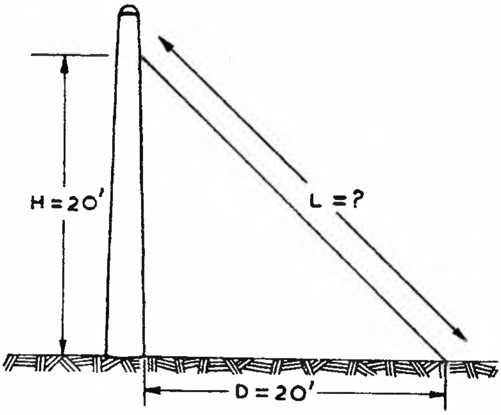
$$= \sqrt{2H^2}$$

$$= \sqrt{2} \times \sqrt{H^2}$$

$$= \sqrt{2} \times H,$$

$$\text{but } \sqrt{2} = 1.414,$$

$$\text{therefore, } L = 1.414 \times H.$$



L = 1.414 x H.

FIG. 11.

Problem. Fig. 11 shows a stay attached to a pole 20' from the ground. The distance from the pole to the point where the stay emerges from the ground is also 20'. What is the approximate length of the stay?

$$L = 1.414 \times H$$

$$= 1.414 \times 20$$

$$= 28.28'$$

$$= \underline{\underline{28'3''}} \text{ (approximately).}$$

8. FINDING POINT OF EMERGENCE OF STAY.

8.1 The point of emergence of the stay is the point on the ground where the stay emerges from its anchorage. Normally the stay forms an angle of 45° with the pole.

8.2 Level Ground. To form an angle of 45° on level ground, measure the height (H) of the pole above ground to the point of attachment of the stay. Then measure the same distance along the ground from the butt of the pole to find the point of emergence of the stay. This is shown in Fig. 12.

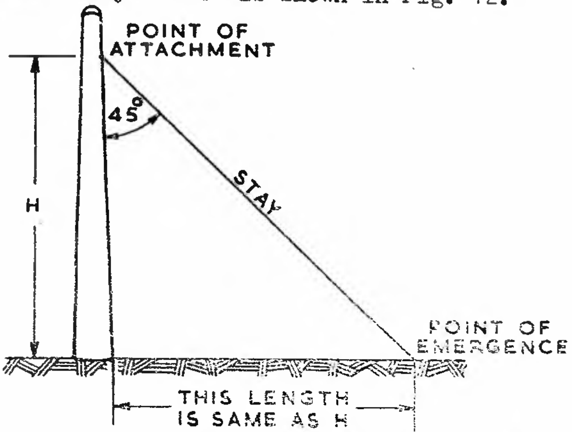


FIG. 12. OBTAINING 45° ANGLE ON LEVEL GROUND.

8.3 Sloping Ground. To form an angle of 45° on sloping ground, the following method can be used. (See Figs. 13a and 13b.)

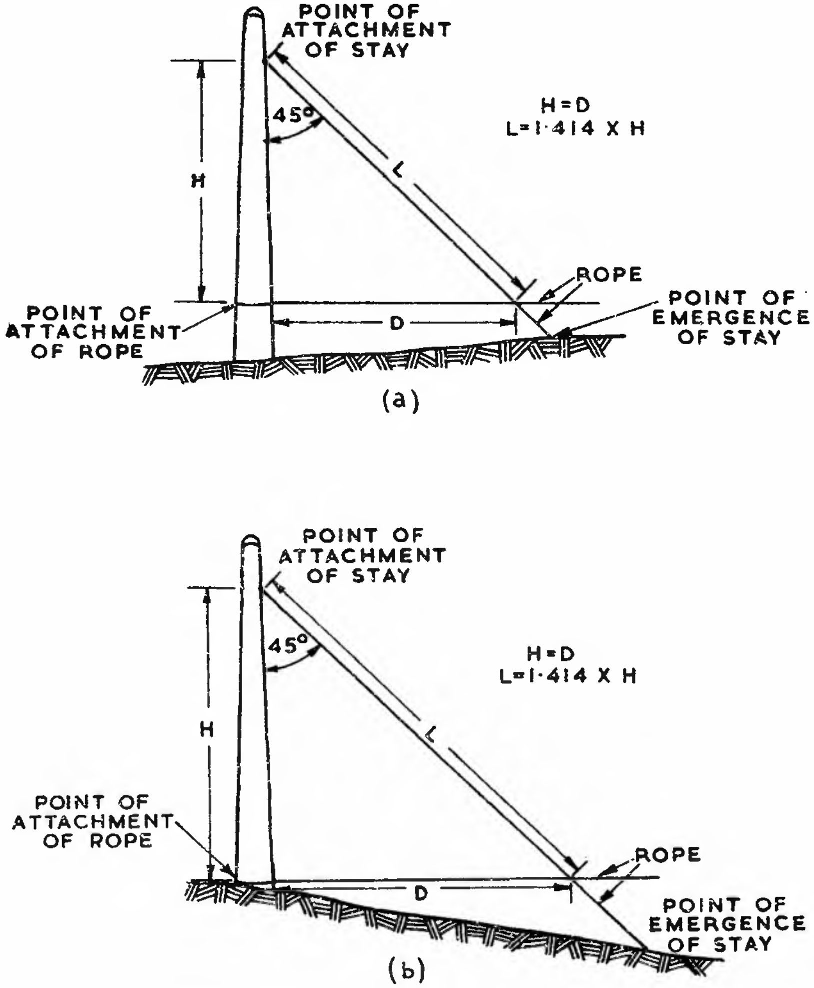


FIG. 13. OBTAINING 45° ANGLE ON SLOPING GROUND.

Attach a length of rope to the pole near the butt or higher if necessary and draw out in the direction of the point of emergence of the stay. Measure the height of the pole from the point at which this rope is attached to the point of attachment of the stay. This is shown as dimension H in Fig. 13. Measure a similar distance along the rope and cut off or mark at this point. This is shown as dimension D.

Another length of rope is secured to the pole at the point of attachment of the stay and this length is marked at a distance L from the point of attachment which equals 1.414 H (see paragraph 7.2).

The two ropes are held taut and the free ends moved until the marks on each rope coincide. The rope representing the stay is extended to touch the ground. This gives the point of emergence of the stay.

9. TEST QUESTIONS.

1. The lengths of the two sides which contain the right angle in a triangle are 5'8" and 4'3". What is the length of the other side?

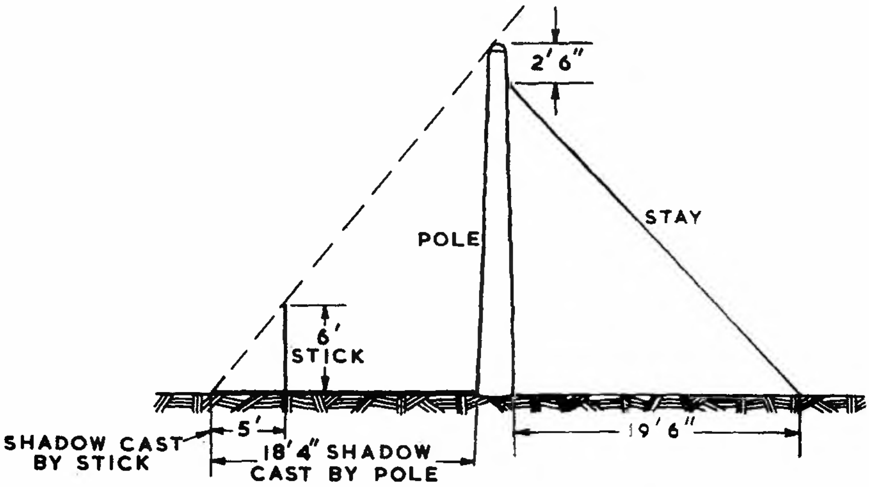
2. Explain a method of finding the line of a stay wire at right angles to a route.

3. Draw a sketch showing how to measure the approximate distance across a river.

4. Explain a method of measuring the approximate height of a pole.

5. Find the approximate length of a stay which is attached to a pole 22' from the ground. The distance from the pole to the point where the stay emerges from the ground is 24'.

6. Find the length of the stay in the Figure -



END OF PAPER.

ANSWERS TO TEST QUESTIONS.

PAPER NO. 1.

1. 1020 insulators.
2. Stock is 7 short.
3. 158 lbs.
4. 118 lbs.
5. 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180.
6. 63,360 ins.
7. 269 yds.
8. 65 poles.
9. 281 poles; 562 arms; 4496 insulators; 112 miles of copper wire.

PAPER NO. 2.

1. 3/4 foot; 1 inch
2. 2.3 inches; 0.05 inch.
3. 8 times; 0.1 inch; 1/10 inch.
4. 0.4 mile.
5. 4%; 7%; 15%; 80%; 97.5%.
6. 20%; 500%.
7. 1 ohm.

PAPER NO. 3.

1. 150 insulators.
2. See Table below.
3. See Section 5.
4. 3-1/8 lbs.
5. 5.68 lbs.
6. 1-1/2 gallons.
7. 20.856 tons per mile.

PAPER NO. 4.

1. (i) $F = \frac{9C}{5} + 32.$
   (ii) $C = 10.$
   (iii) $F = 122.$
2. (i) One 7/12 stay wire, one 5/8" stay rod.
   (ii) One 7/10 stay wire, one 3/4" stay rod.
   (iii) Two 7/12 stay wires, one 1" stay rod.
3. $r = 21.$
4. $L = 28.23.$

PAPER NO. 5.

1. 90°
2. See paragraph 2.3
3. 44 : 7.
4. 33 yds.
5. 3-1/8 sq. yds.
6. 3 cub. yds.
7. 4 trips.
8. 4 gallons.
9. 35-1/2 hrs.
10. 7 hrs.

PAPER NO. 6.

1. 7'1".
2. See Section 4.
3. See Section 5.
4. See Section 6.
5. 32'7".
6. 27'7".

Number	Square	Square Root
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162