THE AUSTRALIAN POST OFFICE

CP 309
PRICE 25c

COURSE OF TECHNICAL INSTRUCTION

Engineering Training Section, Headquarters, Postmaster-General's Department, Melbourne C.2.

BASIC PULSE CIRCUITS.

Page

1. INTRODUCTION.

1.1 This paper examines the effect that simple R-C circuits of various time constants have on pulse waveforms, and shows that the output waveforms from the circuits often differ greatly from the input waveforms. Distortion of the waveform is usually avoided in amplifier circuits where the output must faithfully reproduce the input, but in many pulse circuits the waveforms are purposely distorted or shaped to perform specific functions. The shaping that is introduced by resistors and capacitors is "linear shaping" as opposed to the "non-linear shaping" that can be introduced by circuits containing non-linear components such as diodes.

1.2 The detailed operation of a pulse circuit is usually obtained by considering the charge and discharge of the circuit components. There is, however, an additional way to estimate the output waveforms from some R-C circuits, and this is based on the fact that some circuits produce output waveforms that are very similar to the waveforms produced by the mathematical processes of "differentiation" and "integration". The circuits that approximate these processes are named after the operation that they perform, and since they are common in pulse techniques, a basic understanding of the practical applications of differentiation and integration is important for a simple understanding of the operation of pulse circuits.

1.3 In the paper "Introduction to Pulse Techniques", we saw that circuits containing inductance and resistance behave in ways allied to the behaviour of resistance-capacitance circuits. However, since L-R circuits are not as common in practice as R-C circuits, most of the discussions are related to R-C circuits, and summaries are made of analagous effects in L-R circuits.

2. SQUARE WAVES IN R-C CIRCUITS.

2.1 When a rectangular wave is applied to a circuit, the effect of the circuit on the wave is examined by considering the change of the original wave as a change in D.C. voltage.

In the simplified theoretical circuit of Fig. 1a the changeover switch alternately applies +100V and -100V to the R-C circuit. This is equivalent to applying a rectangular wave (Fig. 1b) with a value of 200V peak to peak, to the R-C components. The timing of the switch in the equivalent circuit determines the repetition rate and the duty factor of the rectangular wave.

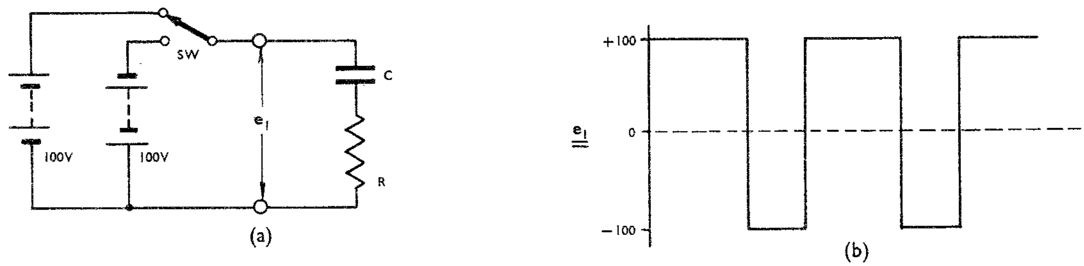


FIG. 1. CIRCUIT PRODUCING RECTANGULAR WAVE INPUT.

The appearance of the waveforms of voltage across the components of an R-C circuit depends on the circuit time constant, with variations depending on whether the time constant is short, medium, or long. These designations are related to the time for one cycle of the wave and are therefore arbitrary. It will be assumed that:

- A short time constant is one in which the charging is completed in 1/10th of the period of the wave, or less.
- A long time constant is one in which the charging is completed in 10 times the period of the wave, or more.
- A medium time constant is between these values.

2.2 Medium Time Constant Circuit. Consider that a square wave with a peak voltage of 100 volts is applied to a circuit with a time constant equal to 1/10th of the period of the square wave (Fig. 2). The circuit is completed at the commencement of a positive half cycle and instantly there is 100 volts across the resistor. The capacitor charges and the resistor voltage (e_R) falls exponentially, and is, for practical purposes, zero after 5 time constants which is a time equal to half of the period of the square wave. At this time, the capacitor is also completely charged to 100 volts.

With the voltage reversal at the commencement of the second half cycle, the voltage present across the resistor is -200V, which is the sum of the capacitor voltage and the applied voltage. The capacitor voltage (e_C) changes exponentially from +100V to -100V, and since there is double the initial voltage acting in the circuit, the initial discharge rate is doubled and the complete change still takes 5 time constants. The resistor voltage decays exponentially to zero in 5 time constants, that is by the end of one cycle of the input square wave.

With the next half cycle, e_R steps instantly to +200 volts and decays exponentially back to zero and e_C changes exponentially from -100V to +100V. The voltages in the circuit are shown in Fig. 2b.

Notice that (except for the initial half cycle) at each transition of the input wave the resistor voltage always steps an amount equal to the peak-to-peak voltage of the input. This is always the case independent of the time constant of the circuit.

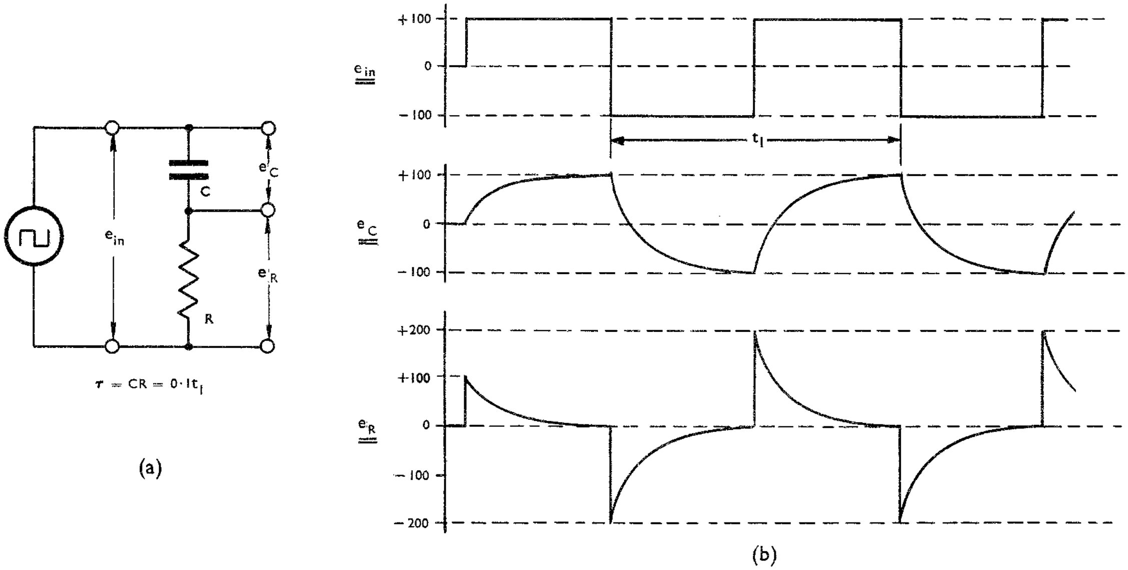


FIG. 2. MEDIUM TIME CONSTANT CIRCUIT.

2.3 Short Time Constant Circuit. When the circuit time constant is made shorter, the capacitor charges more quickly and the voltage across the resistor decreases to zero in a shorter time. In Fig. 3 the time for one half cycle represents 25 time constants and the capacitor is fully charged in 1/10th of the period of the input square wave. The capacitor voltage therefore rises exponentially to reach the maximum in 1/5th of each half cycle of the square wave, and the resistor voltage steps to the maximum equal to the peak-to-peak input signal and returns to zero also in 1/5th of a half cycle. The voltage across the resistor is a series of short "spikes" starting with each transition and the capacitor voltage is still approximately a square wave, but has a definite rise time that is exponential in shape.

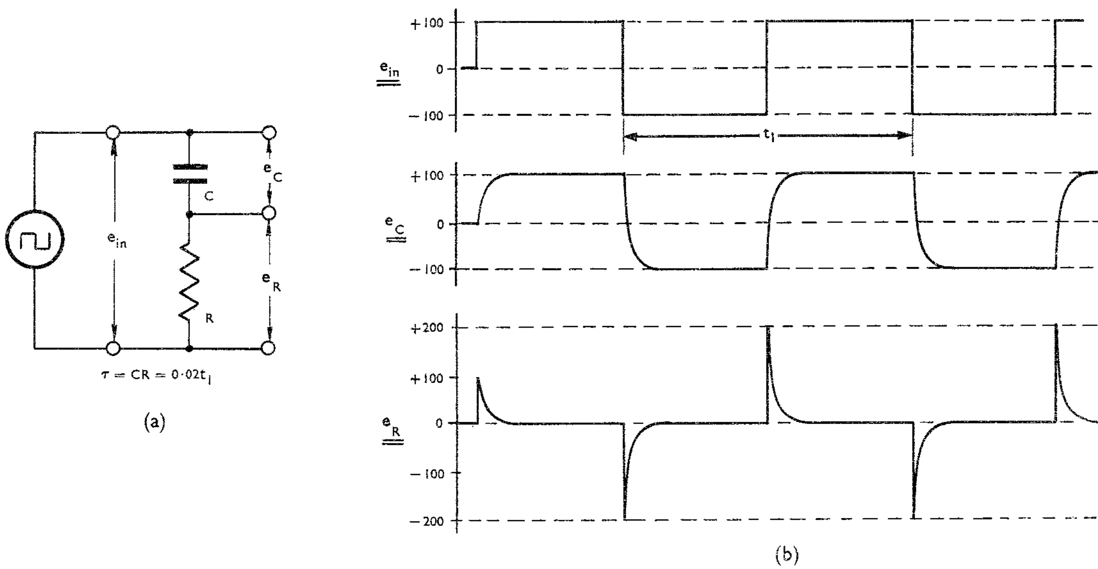


FIG. 3. SHORT TIME CONSTANT CIRCUIT.

2.4 Long Time Constant Circuits. For a long time-constant circuit the charging process is not completed in the half cycle of the square wave input. Consider Fig. 4 where the time for one half cycle of the square wave equals 0.25 time constants and it would take a period equal to 10 cycles of the square wave for the capacitor to fully charge. From tables of exponential functions, in the 0.25 time constants of one half cycle, the capacitor voltage changes by approximately 0.22 of the circuit voltage. In the first half cycle with 100 volts applied the capacitor voltage changes from zero to approximately +22 volts. The resistor voltage at the commencement of the first half cycle steps to +100 volts and then decreases exponentially to approximately 0.78 of the circuit voltage. At the end of the first half cycle the resistor voltage is therefore +78 volts. The changes of voltage are shown in Fig. 4b.

With the applied voltage reversed at the beginning of the next half cycle, the resistor voltage steps by 200 volts from +78 volts to -122 volts. It then decreases to 0.78 of -122 volts and is -95.2 volts at the end of a half cycle. During this half cycle the capacitor voltage changes exponentially from +22V towards -100V. In the half cycle the voltage changes by 0.22 of the maximum possible change of -122V and at the conclusion of the half cycle is -4.8V. In the following half cycle, e_R steps to 104.8 volts and then decreases to 83 volts, while e_C changes to +17 volts.

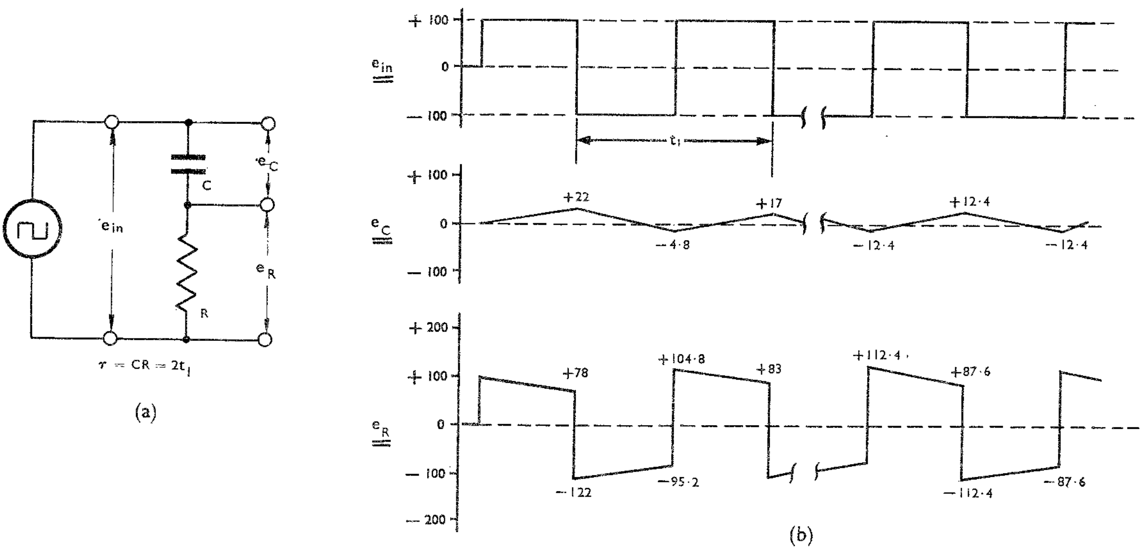
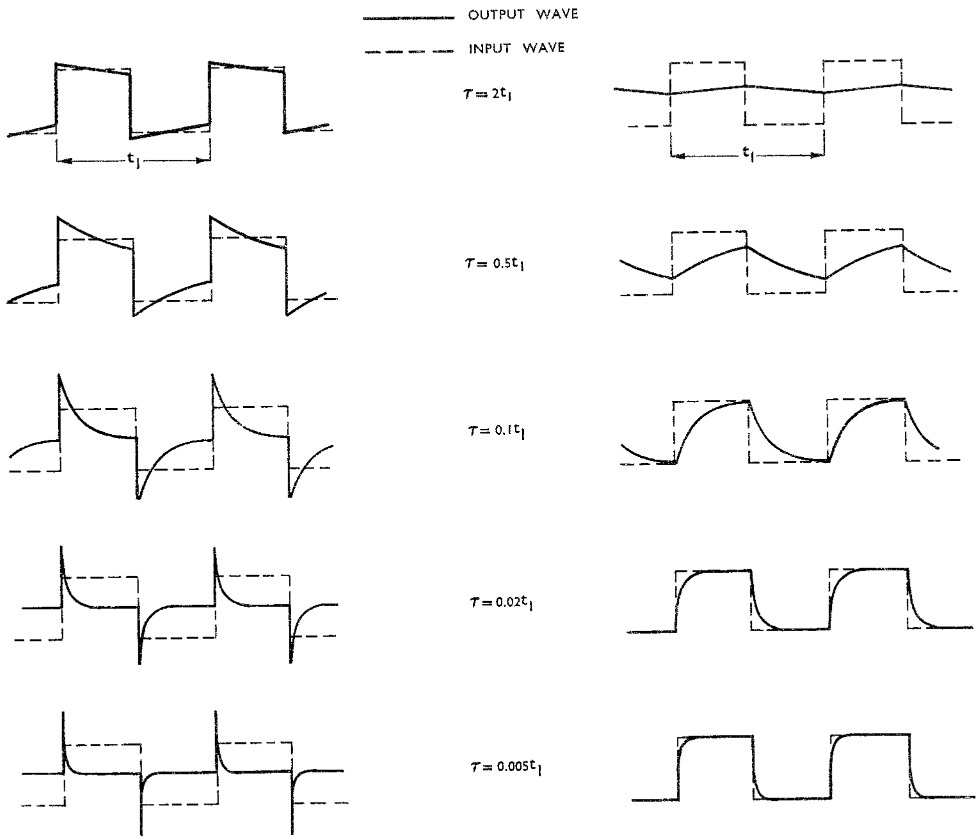


FIG. 4. LONG TIME CONSTANT CIRCUIT.

When the calculations are continued it is found that after a number of cycles the positive and negative half cycles become the same in shape except for the inversion, with e_R stepping to 112.4 volts, and decreasing exponentially to 87.6 volts, and e_C change from -12.4 to +12.4 in one half cycle and back again during the next.

Notice that the voltage across the resistor has an approximately square wave shape, but has a "tilt" on the horizontal sections of the wave. With a square wave input, the values of voltage at the leading and trailing edges of the tilt vary by equal amounts above and below the input voltage respectively. The capacitor voltage is approximately triangular in shape, rising during the positive half cycle and falling during the negative half cycle. Also, though e_C and e_R in Fig. 4 vary along sections of an exponential curve, the variations are limited to the initial (almost linear) section of the curve and between transitions the changes of e_C and e_R are practically linear.

2.5 Comparison of Waveforms. In Fig. 5a the waveforms of the voltages across the resistors of R-C circuits of various time constants relative to the time for a common square wave input, (shown dotted) are compared. When the time constant is very long e_R is practically identical with e_{in}. As the time constant decreases a tilt is produced on sections of the waveform that are normally horizontal. When the circuit time constant is very short, the transitions produce "spikes" of voltage which become shorter in duration as the time constant is shortened. The polarity of the spikes corresponds to the direction of the transitions of the input square wave.



(a) Resistor Voltage Waveforms (e_R). (b) Capacitor Voltage waveforms (e_C).

FIG. 5. COMPARISON OF WAVEFORMS.

The waveforms of capacitor voltage are compared in Fig. 5b. When the time constant is very short e_C is practically identical with e_{in}. As the time constant is increased a "rounding" appears on the latter part of the transitions. With a very long time constant, e_C becomes very close to triangular in shape, with the voltage increasing at an almost constant rate during the positive half cycle of the input wave, and decreasing at an almost constant rate during the negative half cycle. As the time constant is increased e_C approaches closer to the true triangular shape because of the capacitor charge time corresponds to a smaller section of the initial part of an exponential curve.

2.6 Inductor-Resistor Circuits. The preceding waveforms of voltage also apply to L-R circuits without amendment of the time scale. The designations should be altered in each case so that resistor voltage (e_R) is read as inductor voltage (e_L) and capacitor voltage (e_C) as resistor voltage (e_R).

3. SAWTOOTH WAVES IN R-C CIRCUITS.

3.1 In Section 2 and the paper "Introduction to Pulse Techniques" we have examined the charging of a capacitor by step changes of the circuit input voltage. However, sawtooth waves are common in pulse circuits, and this section discusses how a sawtooth wave is affected by R-C circuits. The first consideration is to examine the voltages across the capacitor and the resistor when the voltage starts to increase from zero with a constant rate of change.

3.2 Input Voltage Increasing Constantly. The paper "Introduction to Pulse Techniques" shows that when a "voltage step" is applied via a resistor to a capacitor, the capacitor voltage rises exponentially towards the magnitude of the applied voltage, becoming with time, an increasingly greater percentage of the applied voltage.

Fig. 6 shows that when the applied voltage (e_{in}) is increasing at a constant rate, the capacitor is always attempting to charge to a new higher voltage. The capacitor voltage (e_C) becomes an increasingly greater percentage of the applied voltage as time passes, but can never reach it. The curve of e_C follows a shape as in Fig. 6, and eventually becomes, for practical purposes, parallel to e_{in}.

At the time of commencement of the increase in voltage there is no applied voltage and therefore no circuit current. A short time later when the voltage on the capacitor is only a small fraction of the applied voltage at that instant, the rate of increase of current is high. Later still as the capacitor voltage becomes an increasingly greater fraction of the applied voltage at that time, the rate of increase in current is reduced.

The voltage across the resistor (e_R) is dependent on the current, and therefore the resistor voltage first increases at a rapid rate and then the rate of increase becomes less with time. The initial rate of increase of e_R when the capacitor has no charge is equal to the rate of increase of e_{in}.

The increase of e_R follows an exponential curve (Fig. 6) with a maximum voltage (E_M) equal to the change in the input voltage in one time constant. It can be shown that this is equivalent to the product of the rate of change of input voltage and the circuit time constant.

$$E_M = \frac{de_{in}}{dt} CR$$

where

E_M = Maximum resistor voltage in volts.

$\frac{de_{in}}{dt}$ = Rate of change of input voltage in volts per second.

C = Capacitance in farads.

R = Resistance in ohms.

Therefore when an input voltage varying at the rate of 2 volts per milli-second is applied to an R-C circuit consisting of a 20kΩ resistor and an 0.1μF capacitor in series, the maximum voltage that can be developed across the resistor is:-

$$E_M = \frac{de_{in}}{dt} CR = \frac{2 \times 10^3 \times 0.1 \times 20 \times 10^3}{10^6} = 4 \text{ volts.}$$

Increasing the rate of change of input voltage increases the maximum voltage across the resistor proportionally. When the change of voltage is negative instead of positive, the maximum voltage across the resistor is also negative.

At any instant the sum of the resistor and capacitor voltages equals the input voltage:-

$$e_{in} = e_C + e_R, \quad \therefore \quad e_C = e_{in} - e_R.$$

Since the change of e_R is exponential, after five time constants it is, for practical purposes, constant at a voltage E_M. Therefore after five time constants e_C is separated from e_{in} by a constant voltage, and e_C is parallel to, but delayed from e_{in} by a time equal to one time constant.

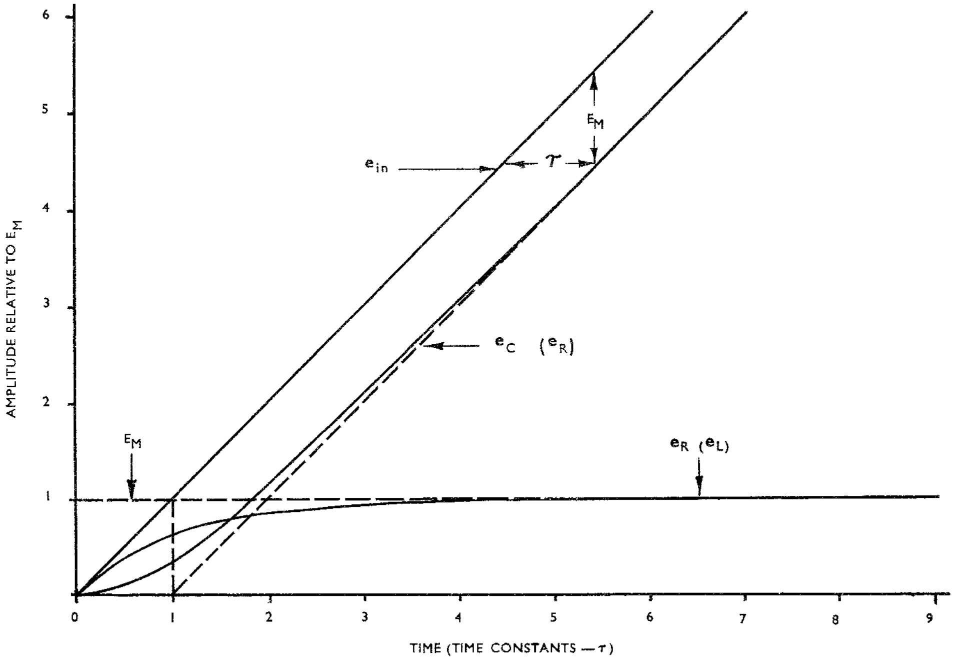


FIG. 6. CONSTANTLY INCREASING VOLTAGE INPUT TO R-C CIRCUITS.

As a matter of interest, following the same reasoning as used to develop the formulae in the "Introduction to Pulse Techniques", the exponential formula for expressing the variation of resistor voltage (e_R) when the input voltage is increasing from zero, is:-

$$e_R = \frac{de_{in}}{dt} CR\,(1 - \epsilon^{-x}),$$

where

e_R = Instantaneous voltage across resistor

ϵ = Base of natural logarithms (2.718)

$x = \frac{t}{CR}$

t = Time in seconds

Other symbols defined on page 6.

3.3 Inductor-Resistor Circuits. In a circuit containing inductance and resistance, when the input voltage commences to rise at a constant rate, the voltage across the inductor increases exponentially towards a maximum, and the resistor voltage (and the current) increases, slowly at first, and then at an increasing rate, eventually approaching a rate of increase equal to the input voltage increase. The curves in Fig. 6 apply to the L-R circuit when the designations are altered so that e_R is read as e_L, and e_C as e_R. The maximum voltage (E_M) across the inductor in the circuit is found with a similar formula to that used for R-C circuits:-

$$E_M = \frac{de_{in}}{dt}\frac{L}{R}$$

where

E_M = Maximum inductor voltage in volts

$\frac{de_{in}}{dt}$ = Rate of change of input voltage in volts per second.

L = Inductance in henries.

R = Resistance in ohms.

3.4 Sawtooth Waves in Medium Time Constant Circuit. The preceding information is now applied to find the voltages in an R-C circuit with a time constant of 1mS when the sawtooth waveform of Fig. 7a is applied. The "forward trace" time represents 20 time constants and the "retrace" time is 5 time constants.

The input voltage changes by 100 volts in a positive direction in 20mS during the forward trace, and by 100 volts in a negative direction in 5mS during the retrace, therefore the rate of change of voltage $\left(\frac{\text{Voltage}}{\text{Time}}\right)$ is:-

$$\frac{100 \times 10^3}{20} \text{ volts/second during the forward trace time.}$$

$$\text{and } \frac{-100 \times 10^3}{5} \text{ volts/second during the retrace time.}$$

The maximum voltage across the resistor during the forward trace ($E_M = \frac{de_{in}}{dt}\tau$) is:-

$$E_M \text{ (forward)} = \frac{100 \times 10^3}{20 \times 10^3} = 5 \text{ volts,}$$

and during retrace is:-

$$E_M \text{ (retrace)} = \frac{-100 \times 10^3}{5 \times 10^3} = -20 \text{ volts.}$$

The resistor voltage during the first half cycle shown increases exponentially and reaches the maximum of 5 volts in 5mS. It then remains constant for the rest of the forward trace. At the commencement of the retrace, the resistor voltage changes exponentially to the maximum of -20 volts in the 5mS retrace time. For the next forward trace, e_R changes exponentially from -20 to +5 volts in 5mS and remains at this value for the rest of the forward trace. The resistor voltage waveform then repeats in the same manner (excepting for the initial half cycle) as shown in Fig. 7 on an expanded vertical scale.

The capacitor voltage is obtained by substracting the instantaneous values of resistor voltage from the input voltage and is shown in Fig. 7c.

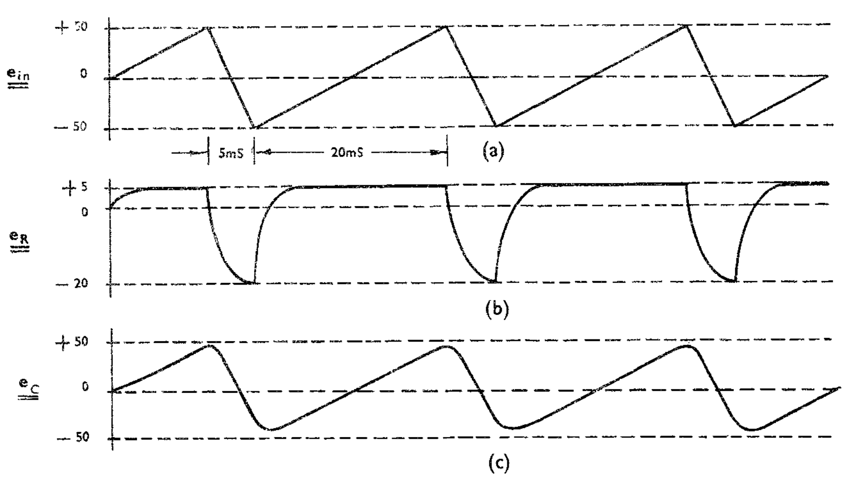


FIG. 7. SAWTOOTH WAVES IN MEDIUM TIME CONSTANT R-C CIRCUIT.

3.5 Sawtooth Waves in Short Time Constant Circuit. Consider now that the same sawtooth wave is applied to a circuit having a short time constant of 0.2mS. Fig. 8 shows the voltages present in the circuit. The resistor voltage reaches a maximum in 1mS, and is therefore constant for the majority of both the forward and the retrace times. It appears as a series of rectangular pulses with exponential transitions of short duration when compared with the pulse duration.

With the same rate of change of input voltage as in Fig. 7, but a different time constant, the maximum voltage during the forward trace ($E_M = \frac{de_{in}}{dt}\tau$) is:-

$$E_M\ (\text{forward}) = \frac{100 \times 10^3 \times 0.2}{20 \times 10^3} = 1 \text{ volt},$$

and during the retrace is:-

$$E_M\ (\text{retrace}) = \frac{-100 \times 10^3 \times 0.2}{5 \times 10^3} = -4 \text{ volts}.$$

Subtracting the resistor voltage from the input voltage to obtain the capacitor voltage gives a waveform (Fig. 8c) that is very close to being a sawtooth but with slightly rounded changes between the forward trace and the retrace.

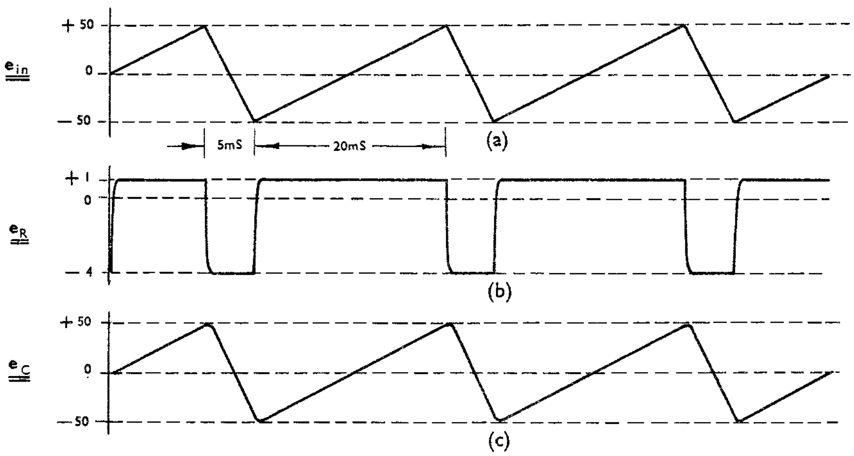


FIG. 8. SAWTOOTH WAVES IN SHORT TIME CONSTANT R-C CIRCUIT.

3.6 Sawtooth Waves in Long Time Constant Circuits. Consider a circuit with a time constant of 50mS. The forward trace of the sawtooth represents 0.4 time constants and the retrace time equals 0.1 time constants.

With the same sawtooth input as in the earlier examples, E_M during the forward trace is:-

$$E_M\ (\text{forward}) = \frac{100 \times 10^3 \times 50}{20 \times 10^3} = 250 \text{ volts},$$

and during the retrace is:-

$$E_M\ (\text{retrace}) = \frac{-100 \times 10^3 \times 50}{5 \times 10^3} = -1000 \text{ volts}.$$

Tables of exponential functions show that in the first 10mS of the input wave in Fig. 9a (0.2 time constants) e_R increases by 0.1813 of the maximum resistor voltage of +250 volts.

e_R after 10mS = 0.1813 × 250 = 45.32 volts.

During the retrace time, the resistor voltage commences to change exponentially from +45.32 volts towards the maximum voltage of −1000 volts. For the retrace 5mS (0.1 time constants), from tables of exponential functions, e_R decreases by 0.0952 of the circuit voltage of 45.32 +1000 volts.

∴ Decrease in e_R during retrace = 0.0952 × 1045.32 = 99.49 volts.

∴ Voltage at end of retrace = 45.32 − 99.49 = −54.17 volts.

In the next complete forward trace of 0.4 time constants, e_R increases from −54.17 volts towards the maximum of +250 volts, and from tables of exponential functions, in 0.4 time constants e_R increases by 0.3297 of the maximum circuit voltage of 54.17 +250 volts.

∴ Increase in e_R during forward trace = 0.3297 × 304.17 = 100.3 volts.

∴ Voltage at end of forward trace = −54.17 + 100.3 = 46.13 volts.

After many cycles the variation of e_R gradually drifts so that it is varying during the forward trace from −52.4 volts to +47.3 volts and back again during the retrace. Fig. 9b shows that the shape of the graph of e_R is not greatly different from the input wave, but a slight curve is introduced onto the transitions, in particular the forward trace.

The voltage across the capacitor (e_C), (found by subtracting e_R from e_{in}), is shown in Fig. 9c on an expanded vertical scale. The waveform shows that when the sawtooth is negative, the capacitor voltage waveform has a negative slope, that is it slopes downward to the right, and when the sawtooth is positive, the slope of e_C is positive. When the sawtooth has a large amplitude either positive or negative, the slope of e_C is great, and approaches closer to the vertical, with the slope decreasing as the sawtooth amplitude decreases. When the sawtooth is zero e_C has zero slope as it is in the process of changing from a positive going section to a negative going section, or vice versa.

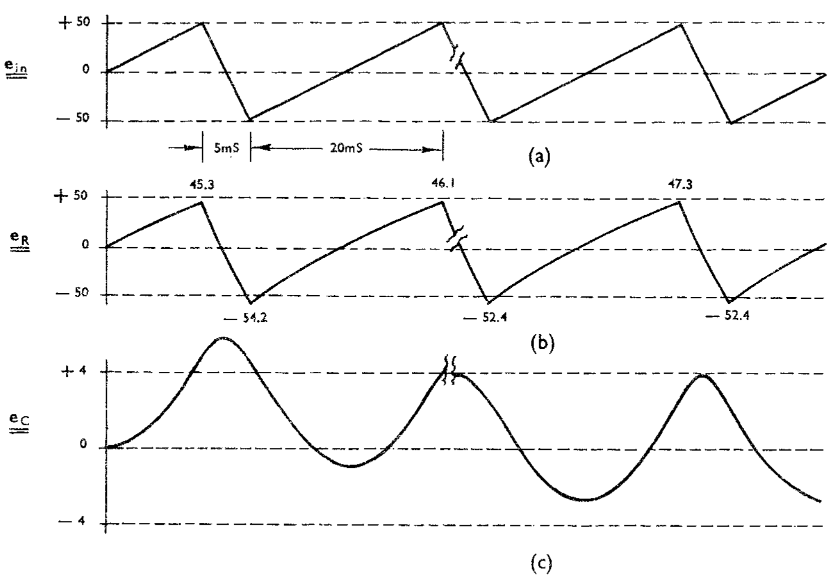
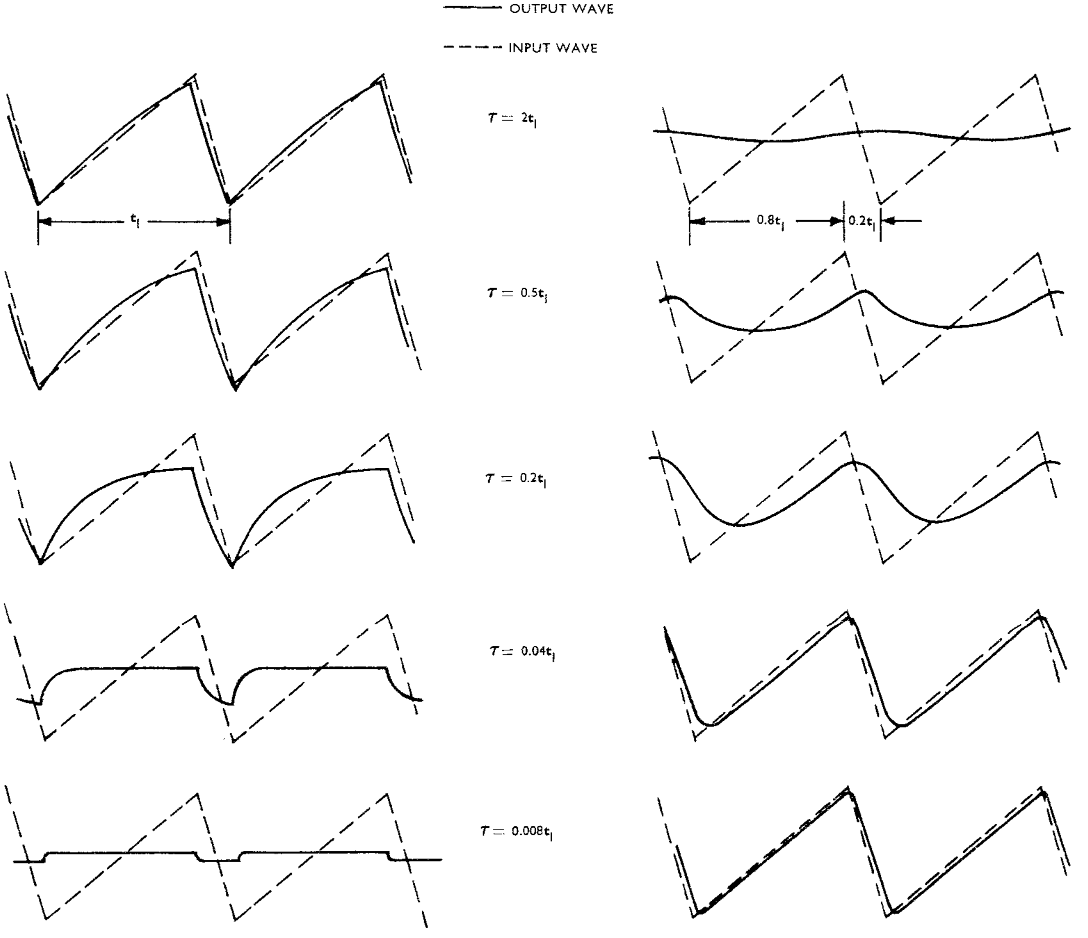


FIG. 9. SAWTOOTH WAVES IN LONG TIME CONSTANT R-C CIRCUIT.

3.7 Comparison of Waveforms. The waveforms of voltages across the resistors of various R-C circuits are compared in Fig. 10a for examples where the input voltage is a sawtooth waveform. For a very long time constant e_R is practically identical with e_{in}. As the time constant is decreased the linear transitions become curved. When the time constant is very short e_R approaches a rectangular wave but has a slight "rounding" of the transitions. The rectangular wave is negative during negative going sections of the sawtooth input and positive during positive going sections. The amplitude of the rectangular wave is greater for the retrace than for the forward trace; that is, the amplitude is greater for the sections of the input wave that have a faster rate of change.



(a) Resistor Voltage Waveforms (e_R). (b) Capacitor Voltage Waveforms (e_C).

FIG. 10. COMPARISON OF WAVEFORMS.

The capacitor voltage waveforms compared in Fig. 10b show that when the time constant is short e_C is practically identical with e_{in} but as the time constant is increased a "rounding" appears at the junction of the forward trace and retrace times. With a very long time constant, e_C is such that its slope at any instant, is dependent on the amplitude of the input sawtooth waveform.

4. COUPLING CIRCUIT.

4.1 When amplifier stages are connected in cascade a "coupling circuit" is often incorporated in the complete amplifier circuit. The purpose of the coupling circuit is to allow the A.C. component of the signal to be transferred from the anode circuit of the first valve to the grid circuit of the following valave, but to prevent interaction between the D.C. supplies providing high tension and bias for the circuit. A commonly encountered circuit in shown in Fig. 11a. An input signal varies the anode current of V_1 and an amplified version of the input signal appears across the anode load resistance (R_1). The coupling capacitor (C) makes the A.C. component of the signal on the anode of V_1 available at the grid of V_2 and across the grid resistor (R_2). The grid resistor is the shunt component of the coupling circuit and provides the D.C. connection to the grid for bias purposes.

The reactance (X_C) of the coupling capacitor increases as the input signal frequency is decreased, and the low frequency components of the signal are attenuated when the capacitive reactance becomes comparable with the circuit resistance. The coupling circuit is therefore an elementary high pass filter. For the circuit to behave as an effective coupling circuit, the reactance of the capacitor at the lowest frequency to be amplified must be very small by comparison with the circuit resistance.

This section examines the behaviour of the coupling circuit from the pulse point of view. An analysis to relate the time constant of the coupling circuit with its amplitude-frequency and phase-frequency characteristics is included in Section 3 of the paper "Video Test Signals".

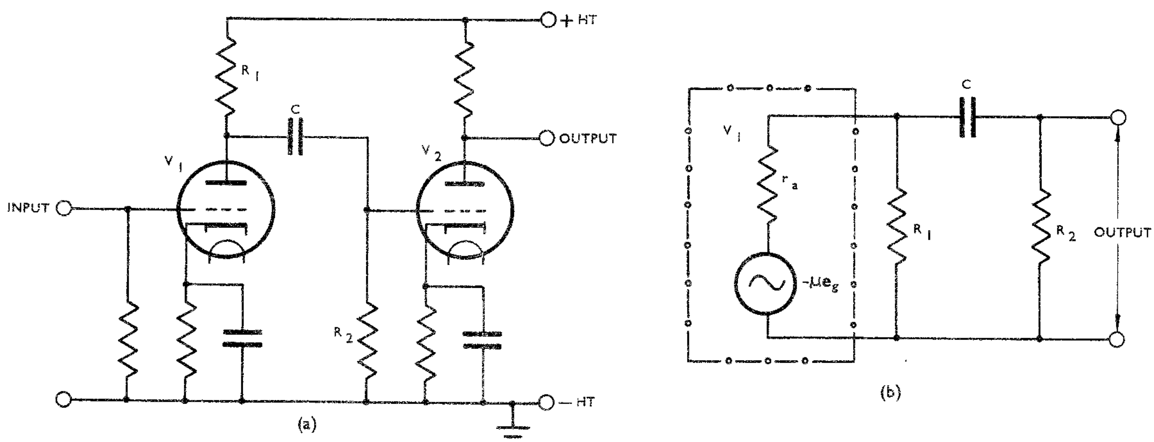


FIG. 11. AMPLIFIER AND COUPLING CIRCUIT.

A low frequency equivalent circuit of a valve and coupling circuit is shown in Fig. 11b. The coupling circuit is fed from a source which includes the anode resistance of the valve and its load resistance. In many cases, particularly in pulse applications, the equivalent resistance of the valve circuit is low by comparison with the value of the grid resistor of the coupling circuit and the signal to the coupling circuit can be considered as coming from a zero impedance source.

4.2 Time Constant. The object of a coupling circuit is to transfer the input signal to the output with as little change of shape as is possible. The output waveform from a coupling circuit is taken across the resistor (Fig. 12).

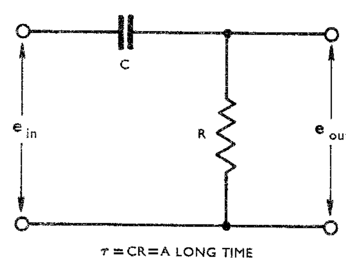


FIG. 12. COUPLING CIRCUIT.

Examination of the resistor waveforms for a square wave input as shown in Fig. 5a indicates that as the circuit time constant is increased, the charge on the capacitor takes longer to vary, and the output waveform becomes closer to being a replica of the input waveform. Note that a coupling circuit works by virtue of the face that the capacitor does not charge and discharge. When the circuit time constant is infinitely long, no change of capacitor charge occurs and the input voltage is coupled, without modification, to the output.

Practical circuits introduce a "tilt" on the normally horizontal sections of rectangular waves. The amount of tilt is dependent on the time constant of the circuit by comparison with the duration of the sections of the wave. The tilt introduced by a circuit is usually expressed in terms of the tilt introduced onto a square wave input.

4.3 Square Wave Tilt. The tilt present on a square wave is measured as the difference in level between the initial and final levels of the square wave relative to the amplitude of the transitions and is expressed as a percentage. This is shown in Fig. 13, where:-

$$\text{Tilt} = \frac{E_1 - E_2}{E_1 + E_2} \times 100\%.$$

The tilt introduced onto a square wave by a single coupling circuit is related to the time for a half cycle of the square wave and the time constant of the coupling circuit.

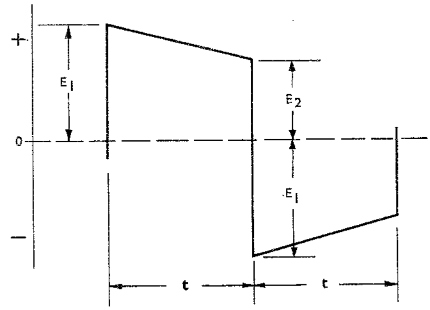


FIG. 13. TILT.

The output amplitude decays exponentially from E_1 to E_2, therefore:-

$$E_2 = E_1\, \epsilon^{-x} \qquad (\text{where } x = \frac{t}{CR}).$$

$$\therefore \text{Tilt} = \frac{E_1 - E_1\,\epsilon^{-x}}{E_1 + E_1\,\epsilon^{-x}} \times 100\% = \frac{1 - \epsilon^{-x}}{1 + \epsilon^{-x}} \times 100\%.$$

When the tilt is small as is normal in practice, the exponential decay from E_1 to E_2 is approximately linear and varies at the initial rate of change of the exponential curve. By making the approximation that the tilt is linear, it can be shown that:-

$$\textbf{Tilt} = \frac{50t}{CR}\ \%$$

This gives little error for values of tilt up to approximately 10% and is adequate for most practical examples up to approximately 20%. The following example illustrates the validity of this approximation.

Example: Find the tilt produced on a 50c/s square wave by a coupling circuit consisting of an 0.1μF capacitor and a 1MΩ resistor.

$$x = \frac{t}{CR} = \frac{0.01 \times 10^6}{0.1 \times 10^6} = 0.1.$$

From tables of exponential functions, when $x = 0.1$, $\epsilon^{-x} = 0.9048$.

$$\text{Tilt} = \frac{1 - \epsilon^{-x}}{1 + \epsilon^{-x}} \times 100\% = \frac{0.0952}{1.9048} \times 100\% = 4.997\%.$$

By approximation,

$$\text{Tilt} = \frac{50t}{CR}\,\% = 50 \times 0.1\% = 5\%.$$

Answer: Tilt produced by coupling circuit = 5%.

4.4 D.C. Component of Signals. In the long time constant circuit of para. 2.4 (capacitor fully charged in 10 complete cycles of the input signal), when the input is first applied, the voltage across the resistor steps in the positive direction to +100 volts, and for a number of cycles the wave gradually changes its position in the vertical direction, and finally becomes symmetrical about the zero axis. This waveform is reproduced (to a different scale) in Fig. 14b. As the final waveform is symmetrically disposed about the zero axis, the area above the axis equals the area below the axis and so there is no D.C. component.

Although there is no D.C. component contained in the input signal, a displacement of the average axis of the wave is produced for a period after the completion of the circuit. The amount of this displacement depends on the voltage present when the input is connected, and the final repetitive amplitude of the output signal. The average axis of the output signal is included in Fig. 14b and this axis decreases exponentially from its initial value towards zero.

Independent of the type of input waveform, this shift of the average axis occurs with any circuit containing reactive components which cause some modification to the amplitude or shape of the input signal, if the input circuit is completed at a time when some input voltage is present. In audio circuits the exponential change of the average is too low in frequency to be heard. In video circuits, if no special prevention circuits are included, switching can cause a temporary shift in the black level of the picture which will be incorrectly reproduced for the period of the change. It takes a time approximately equal to five times the circuit time constant for the average of the output signal to decrease to less than 1% of its initial value, but the duration of the effect in practice depends on the relative amplitudes of the initial average voltage and the peak-to-peak amplitude of the signal. When the input signal contains a D.C. component with a large amplitude compared with the A.C. component of the signal, the initial amplitude of the average axis of the output is large, and it takes a long time for the average to reduce to a small percentage of the A.C. component of the signal.

In Fig. 14d the output waveform from the same coupling circuit is shown when the input voltage is a square wave with a D.C. component (Fig. 14c). The voltage initially steps to +300V when the input is first applied, and again the average axis decays exponentially towards zero. However in this example it takes considerable time for the average to become a negligible proportion of the output signal amplitude. As in the previous example, after the stabilizing of the axis, no D.C. component is present in the output. The coupling circuit has removed the D.C. component of the input signal. After its initial charging, the capacitor maintains an average voltage due to the charge stored, equal to the D.C. component of the input signal, which in this case, by inspection, is +200 volts.

Consider that a rectangular waveform with a pulse duty factor of 0.2, and with a D.C. component as shown in Fig. 14e, is fed into the coupling circuit. The D.C. component of this input wave is +140 volts. The A.C. component of the input waveform, therefore, is a rectangular wave varying from -40 volts to +160 volts.

The output waveform from the coupling circuit is shown in Fig. 14f and once again includes an exponential decay of the average of the wave from its initial value. Also the final wave, even though a tilt is introduced, is very close to being the same as the A.C. component of the input signal, proving that the D.C. component has been removed.

Important points to notice about the waveforms following switching transients are:

- The average axis decays exponentially from its initial value towards zero.

- The waveform at any point is the algebraic sum of the average and the final repetitive waveform.

This information can be used for convenient calculation of the details of the waveforms following a switching transient when such calculations are necessary.

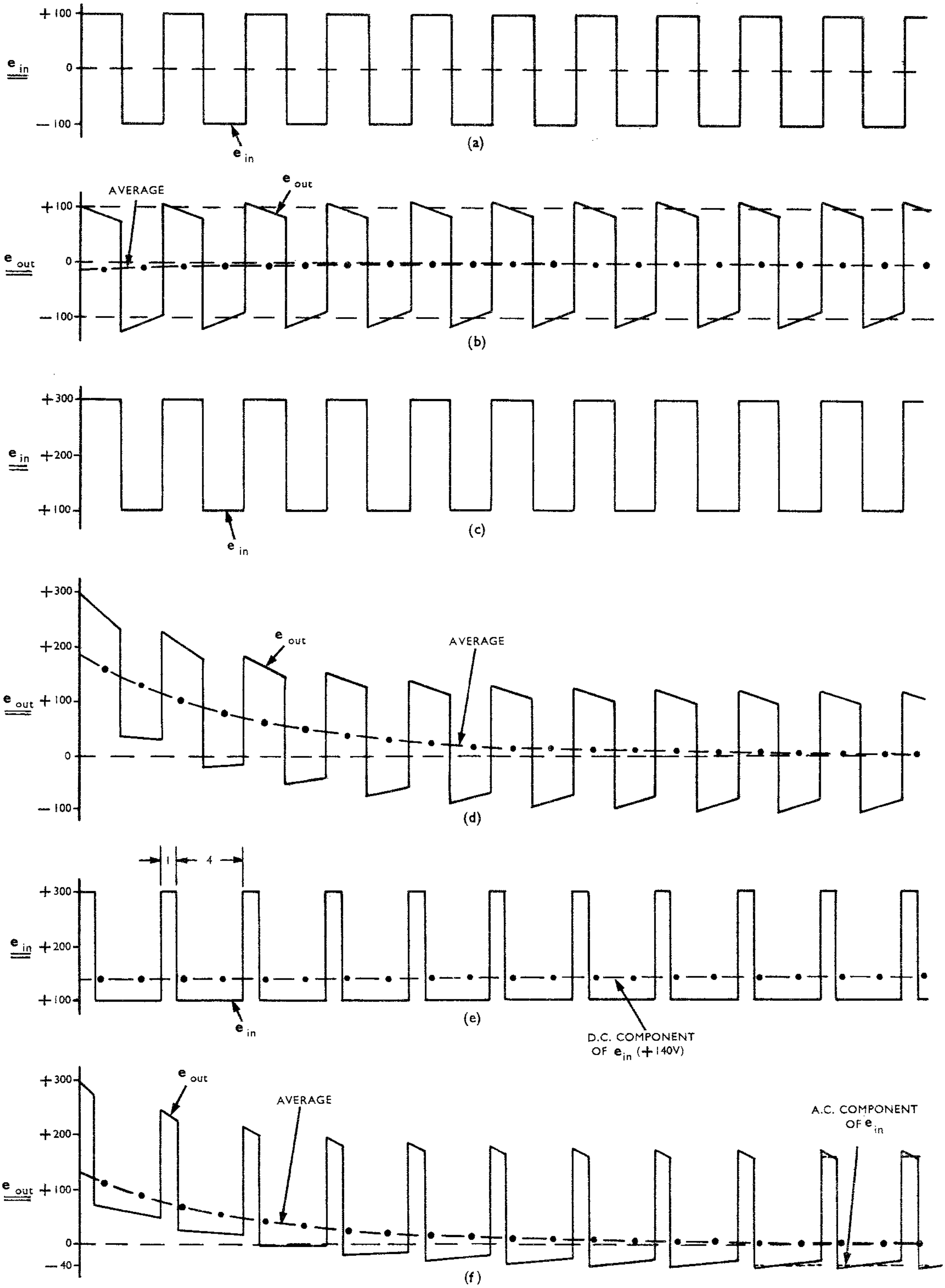


FIG. 14. COUPLING CIRCUIT OUTPUT WAVEFORMS.

5. HIGH FREQUENCY LOSS CIRCUIT.

5.1 Shunt capacitance inevitably exists in all practical circuits, and is a practical problem. Together with any source resistance and other series resistance in the circuit, the capacitance and resistance form a circuit as in Fig. 15, where the output voltage is taken across the capacitor.

An examination of the capacitor voltage waveforms in Fig. 5b shows that, with a square wave input, the output voltage across the capacitor of an R-C circuit with a short time constant has the transitions modified so that they are exponential in shape. When the time constant of the circuit is made shorter, the output waveform approaches closer to the ideal. Therefore, to prevent major degradation of the input waveform, the circuit must be arranged so that the time constant is very short.

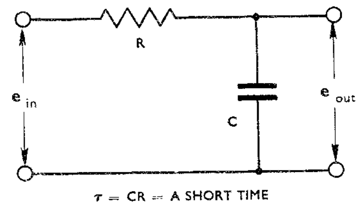


FIG. 15. HIGH FREQUENCY LOSS CIRCUIT.

The rise time of the exponential transitions of the output waveform is indicated relative to the circuit time constant (τ) in Fig. 16. It takes 0.1τ for the output voltage to reach 10% of the maximum amplitude, and 2.3τ to reach 90% of the maximum amplitude, therefore the rise time of the output square wave is 2.2 times the time constant of the circuit.

Rise Time = 2.2 CR

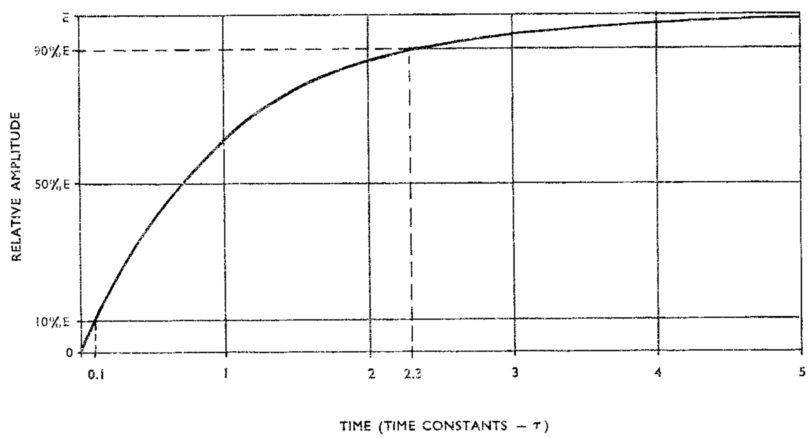


FIG. 16. RISE TIME.

5.2 From an amplitude-frequency response point of view, as the frequency is increased, the reactance of the shunt capacitor decreases, and in conjunction with the series resistance, this causes attenuation of the high frequencies. The circuit then is an elementary low pass filter. An analysis to relate the amplitude-frequency and phase-frequency characteristics of the circuit to the rise time of output square waves, is included in Section 3 of the paper "Video Test Signals".

5.3 The practical considerations to reduce the degradation of the output waveform entail keeping the shunt capacitance as low as possible and then reducing any series resistance, including any source resistance, until the bandwidth of the circuit, and the output waveform, is satisfactory. This is the initial consideration when designing wideband amplifiers where the anode load resistance must be made low to achieve sufficient bandwidth for pulse amplification.

6. DIFFERENTIATION AND INTEGRATION.

6.1 In preceding sections of this paper we have considered the effect of R-C circuits on waveforms by considering the charge and discharge of the capacitor via the resistor and this is quite convenient for waveforms with simple shapes. When the circuit causes minor degradation, as does a coupling circuit or the inherent shunt capacitance in practical circuits, its effect can usually be estimated, even for complex waveforms, by using a knowledge of the circuit time constant to indicate the introduction of tilt or increase in rise time. In pulse circuits, however, the component values are often chosen to shape the waveforms so that they bear little resemblance to the original input and, with complex input waves, estimation of the output waveform by considering charge and discharge becomes difficult.

Pulse shaping using linear components occurs because the values chosen for these components modify the amplitude and phase relationships of the frequencies forming the input pulses. Shaping is required to allow electronic circuits to recognise and make use of certain properties of the input waveform, and to produce special purpose waveshapes from available or more easily generated waveforms.

As mentioned in the introduction, some circuits produce output waveforms that simulate the waveforms produced by mathematical processes known as "differentiation" and "integration". These processes are encountered in the branch of mathematics known as "calculus". The circuits concerned fall into the category of circuits that produce major changes in the shape of the input waveform and these circuits are names after the processes that they approximate. An estimation of the output waveform is often more conveniently obtained from a basic knowledge of calculus than from a detailed study of the charge and discharge of components.

This section introduces the basic concepts of calculus. Following sections extend these ideas to waveforms and compare these with practical differentiating and integrating circuit waveforms. A graphical approach is used, as an algebraic approach is beyond the scope of this course.

6.2 Differentiation. Differentiation is the process of finding an amplitude that is proportional to the rate of change of the original quantity. As rate of change is the ratio of the change of the dependent variable on the "Y" axis of a graph resulting from a change of the independent variable on the "X" axis, the relationship shown by the graph is said to be differentiated with respect to the independent variable. A waveform illustrates voltage variation with time and is differentiated with respect to time. The resultant, which can be drawn as another graph or waveform, is called the derivative of voltage with respect to time. The term "with respect to time" is understood when considering waveforms and is often "dropped". Since the derivative is proportional to rate of change, differentiation "exaggerates" rapid transitions of the original waveform and "loses" sections of constant amplitude.

6.3 Integration. Integration and differentiation are inverse processes. Since the amplitude of the derivative is dependent on the rate of change of the original, integration is the process of producing a resultant with a rate of change proportional to the amplitude of the original. By integrating voltage waveforms with respect to time the resultant obtained is known as the integral of voltage with respect to time and the effect of integration is to produce waveforms that "resist" or "smooth out" changes that appear in the original waveform.

6.4 Rate of Change. When a graph is straight the rate of change is the same throughout its length, but with a curved graph the rate of change varies from point to point. The rate of change or slope of a curve can be found by calculating the ratio of measured small changes in both variables of the graph. However, this can only be accurate when the increments of the variable are infinitely small and therefore not conveniently measured. The difficulty can be avoided by drawing a tangent to the curve at the point of interest and then by determining the rate of change of the tangent using increments of convenient size. Since the tangent is straight its rate of change is constant and is accurately the rate of change of the curve at the point of contact.

6.5 Examples of Differentiation. The relationship between the original graph and the resultant produced by differentiation can be examined by considering simple examples. The graph in Fig. 17a (full line) shows the variation of distance travelled by a car, against time. The graph is a straight line and the increase in distance is the same for each unit of time. Therefore the rate of change of distance with respect to time, which we know as speed, is constant at 40 miles per hour (Fig. 17b). If the distance that the car travelled was as shown dotted in Fig. 18a, the rate of change of the graph is greater as the car is travelling a greater distance per unit of time. As the graph is still a straight line the speed is again constant but is greater (60 m.p.h.). The speed of the car at each instant is shown graphically in Fig. 17b.

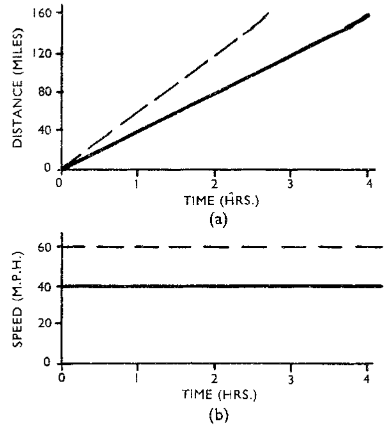


FIG. 17. DIFFERENTIATION OF A LINEAR CHANGE.

The conclusion is that the derivative of a graph increasing linearly is of constant amplitude, the amplitude being dependent on the rate of increase. When the graph is decreasing at a constant rate, the slope of the graph is considered as negative and the derivative is a graph with a constant negative amplitude.

Consider now the graph in Fig. 18a, of the distance travelled by a car with time. The rate of change of the graph is very slow near zero, but as time passes the rate of change increases. The derivative of distance with respect to time is therefore increasing, and the car is accelerating.

In Fig. 18b the derivative (speed) is shown increasing at a constant rate. This will be the case when the graph of distance against time is "parabolic" in shape, in which case the distance is proportional to the square of the time. The conclusion then is that the resultant produced by differentiating a parabolic curve is a straight line with a constant rate of change.

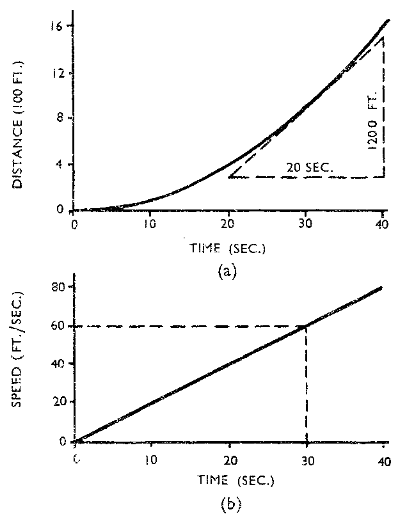


FIG. 18. DIFFERENTIATION OF A PARABOLIC CURVE.

The most convenient way to find a numerical value for the rate of change of a curve is from a tangent to the curve at the point of interest. In Fig. 19a, a tangent is drawn to the curve at a point corresponding to a time 30 seconds from the start. For a time increment of 20 seconds (20 to 40 seconds) the increment of distance is 1,200 feet and the rate of change at 30 seconds is 60 ft. per sec.

Consider now that the car is stopped. This is indicated on a graph (Fig. 19a) as a line at a constant distance from the start at all time. The line has zero rate of change which is independent of the amplitude of the constant (the distance the car travelled). Therefore the derivative of a graph with a constant amplitude is zero (Fig. 19b).

6.6 In summary, the process of differnetiation is the process of finding the rate of change of a graph for all values on the X axis, the resultant being represented graphically.

After differentiation:

- A parabolic curve becomes a graph with a constant slope or a linear change.
- A graph with a constant slope becomes a graph with a constant amplitude.
- A graph with a constant amplitude becomes a line with no amplitude.

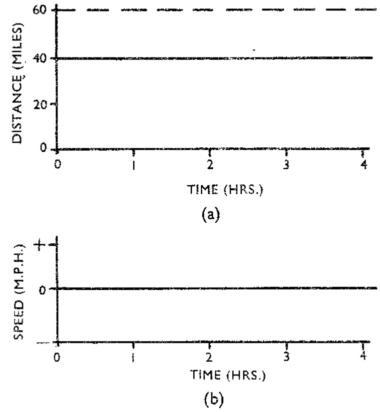


FIG. 19. DIFFERENTIATION OF A CONSTANT.

6.7 Examples of Integration. Since by integration the rate of change of the resultant is dependent on the amplitude of the original, when a graph with a constant amplitude is integrated the resultant has a constant rate of change. The rate of change of the resultant is large when the original amplitude is large and the resultant rate of change is positive when the amplitude is positive and vice versa.

By integrating a graph with an amplitude that is always changing, the rate of change of the resultant is always changing, and for an original graph having a constant rate of change the resultant produced is parabolic in shape.

As the differentiation of any constant amplitude gives zero as the resultant, so integration of zero can produce any constant as a resultant. This constant can appear as an extra consideration when any graph is integrated, and more information is required before the actual value of the constant can be determined. In the simplest form this information would be that the resultant produced by integration starts from zero.

Resultant graphs produced by integration are illustrated in Fig. 20. The integral of each graph is the graph immediately above it. Notice that the inverse is also true; that is, the derivative of a graph is the graph immediately below it.

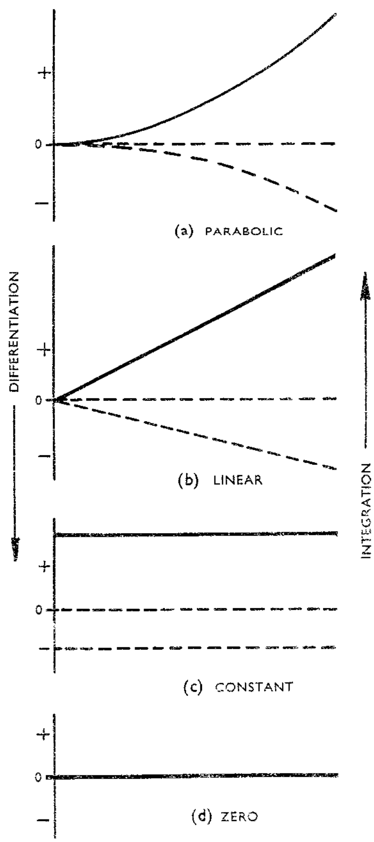


FIG. 20. INTEGRATION.

6.8 Summation of Areas. The original application of integration was to find the area between a curve and the zero axis when it was not possible to do this by geometric means. This principle can often be applied to estimate the integral of the original graph. The process of integration is the progressive summation of the areas of an infinitely large number of infinitely narrow strips enclosed between the curve and the "X" axis.

Consider that the graph in Fig. 21a, showing a constant power in a circuit at all values of time, is divided into narrow strips of equal width so that each rectangle has the same area. Starting from zero the areas will be progressively added. At zero the area is also zero. After one second the area is equal to one "unit". After another equal time, the total area equals two "units". After a further equal time, making a total of 3 seconds, the area equals three "units". These points are plotted in Fig. 21b and they show that, with the passage of time, the addition of areas of equal size makes the total area increase at a constant rate. This coincides with the resultant produced by considering integration as the inverse of differentiation.

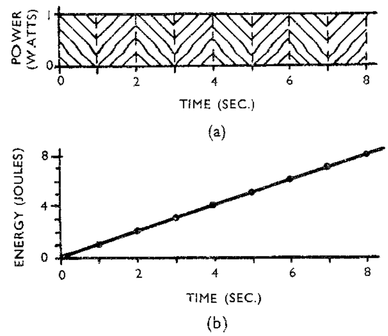


FIG. 21. INTEGRATION BY SUMMATION.

In making the summation to find the area, the smaller the increments of time, the narrower are the strips and the more detailed and accurate is the information about the shape of the resultant. This is particularly important when other examples are considered where the amplitude of the original is not constant. When the increments of time are infinitely small, the resultant is accurately the area enclosed by the curve at any time from the starting point, and this resultant is the integral of the quantity (power) on the "Y" axis with respect to the quantity (time) on the "X" axis.

Note that the summation of all of the instantaneous values of power in a circuit over a time is a measure of energy. The kilowatt-hour meter used to determine charges for commercial electricity supplies indicates total energy by taking the sum of the instantaneous powers at all instances over a period of time. The indication is therefore the integral of a graph showing the power in the circuit at each instant.

6.9 Other Applications. In addition to the analysis of waveforms in R-C circuits, calculus has many other applications in the examination of electrical circuits, particularly from a mathematical point of view. Many of these applications are familiar, though not necessarily recognised as being related to differentiation and integration. The voltage across an inductance (e_L) depends on the rate of change of current through it. Therefore e_L is proportional to the derivative of inductor current (i_L) with respect to time, and the inverse, i_L is proportional to the integral of e_L with respect to time. Other examples are:

- Power is the rate at which energy is expended; therefore power is the derivative of energy with respect to time.

- Current is the rate of change of charge or quantity of electricity; therefore current is the derivative of charge with respect to time and charge is the integral of current with respect to time or the sum of all of the instantaneous values of current over a period of time.

- Voltage across a capacitor (e_C) is proportional to the charge, but charge is the integral of current with respect to time; therefore e_C is proportional to the integral of current with respect to time.

It is not necessary for "time" to be the quantity on the "X" axis of a graphical representation. Examples are graphs showing characteristics of valves. In stating the characteristics of a valve numerically, the mutual conductance is often specified for a particular value of anode voltage and anode current. Mutual conductance (g_m) is the ratio of a small change in anode current (I_a) resulting from a small change in grid voltage (E_g) with the anode voltage (E_a) maintained constant. This gives the slope of the grid voltage/anode current (mutual) characteristic and so g_m is the derivative of I_a with respect to E_g for specific operating conditions. Similarly, amplification factor (μ) is the derivative of E_a with respect to E_g, and anode resistance (r_a) is the derivative of E_a with respect to I_a.

7. DIFFERENTIATION.

7.1 Differentiation of a Square Wave. The principles outlined in Section 6 can readily be applied to find the resultants produced by differentiating voltage waveforms with respect to time. Consider that a square wave (Fig. 22a) is differentiated. The wave produced depends on the rate of change of the initial wave, therefore, during the horizontal sections between B and C there is no rate of change and no amplitude. During the transitions the rate of change is infinite, and theoretically the amplitude after differentiation is also infinite. The transition AB is positive going, that is its rate of change is positive, and it produces a positive pulse on the resultant wave. Between C and D the negative transition produces a negative pulse.

The resulting wave (Fig. 22b) produced by differentiating a square wave is a series of pulses, of infinite amplitude and no duration, and with polarities corresponding to the directions of the changes of the transitions.

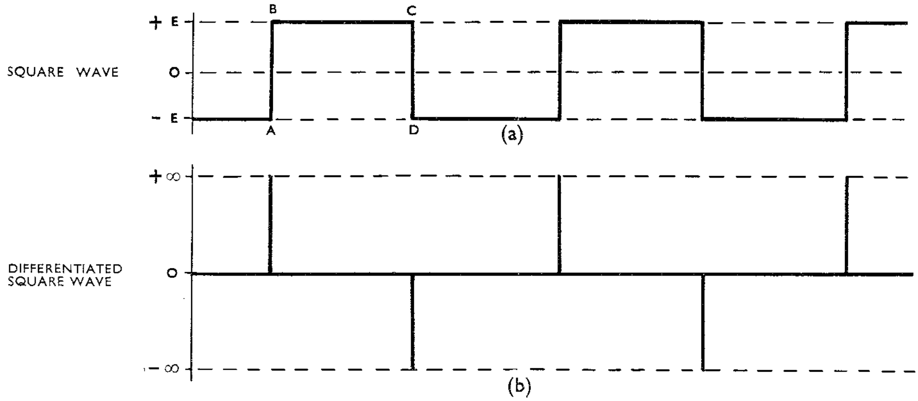


FIG. 22. DIFFERENTIATION OF A SQUARE WAVE.

7.2 Differentiation of a Sawtooth Wave. The waveform resulting from differentiating a sawtooth wave is shown in Fig. 23b. For the forward trace between A and B (Fig. 23a) the waveform has a constant positive rate of change, therefore the resultant waveform after differentiation has an amplitude that is constant and positive.

During the negative going linear retrace period (between B and C), the resultant waveform produced by differentiation has a constant negative amplitude, but as the rate of change during retrace is greater than during the forward trace, the negative section of the resultant has a greater amplitude than the positive section.

Therefore differentiating a linear sawtooth wave produces a rectangular wave with amplitudes dependent on the rate of change of the sawtooth wave.

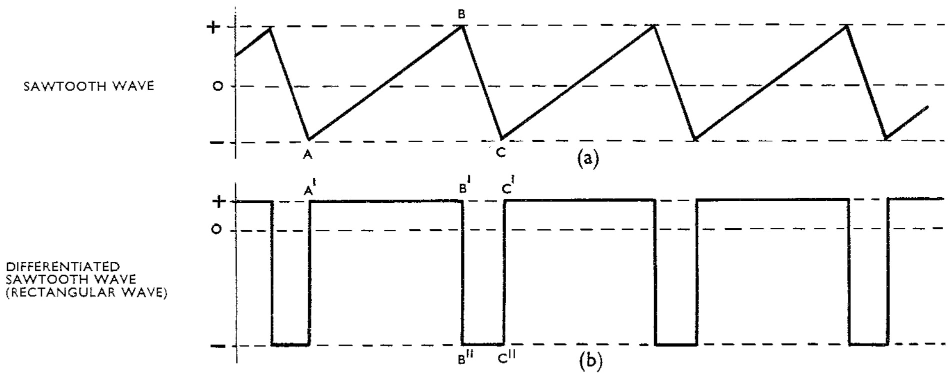


FIG. 23. DIFFERENTIATION OF A SAWTOOTH WAVE.

7.3 Differentiation of a Parabolic Wave. A parabolic wave, so named because it consists of sections of parabolas, is shown in Fig. 24a. The rate of change of the wave is always varying. At point A the rate of change is great and negative going. Approaching point B, the rate of change gradually reduces until at B it is zero. From B to C the wave is changing in a positive direction with an ever increasing rate of change. Differentiation of this wave then produces a maximum negative amplitude at A' (Fig. 24b), gradually decreasing to zero at B' and increasing to a positive maximum at C'.

Between C and E the positive rate of change reduces quickly to zero at D and changes to a maximum negative rate of change again at E, to give the rapid variation of the differentiated wave from the positive maximum (C') through zero (D') to the negative maximum (E').

Differentiating a parabolic wave produces a sawtooth wave as is shown in Fig. 24b.

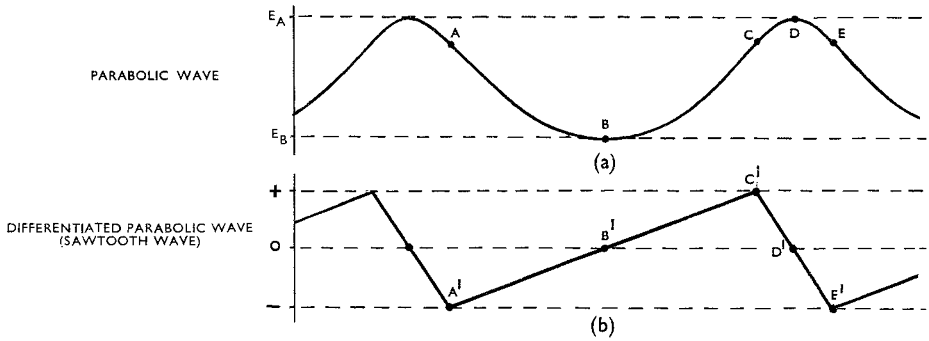


FIG. 24. DIFFERENTIATION OF A PARABOLIC WAVE.

7.4 Differentiation of a Sine Wave. For a sine wave (Fig. 25a), the maximum rate of change occurs as the wave passes through the zero axis. The amplitude of the wave produced by differentiation is therefore a maximum at this time, being a maximum positive at point A' and a maximum negative at point C' (Fig. 25b). At the peak of the sine wave the rate of change is zero and so the amplitude of the resultant wave at this time is also zero (B' and D').

The wave produced by differentiating a sine wave is a cosine wave which has the same shape as a sine wave but is leading the sine wave by 90°.

Applying the same reasoning, the cosine wave when differentiated becomes an inverted sine wave (Fig. 25c). Differentiating cosine θ gives -sine θ.

The rate of change as a sine wave passes through zero is such that, when the slope is continued, the amplitude reaches the peak value of the wave in one radian ($= \frac{1}{2\pi}$ cycles = 57.3°). This is shown in Fig. 26a.

When the sine wave in Fig. 26a is differentiated with respect to time where the unit of time is the time for one radian (t_1), the cosine wave produced (Fig. 26b) has an amplitude equal to the sine wave in Fig. 26a.

Consider a sine wave (Fig. 26c), with half the frequency of the original wave. When it is differentiated with respect to time, and the original unit of time (t_1) is maintained, the cosine wave resulting (Fig. 26d) has the same frequency as the sine wave in Fig. 26c, but has only half the amplitude. The reason is that when the frequency is halved, the maximum rate of change of the sine wave is halved, and reaches the maximum sine wave amplitude in two radians ($2t_1$) of the original sine wave. Therefore, by differentiating sine waves of various frequencies with respect to time, the amplitudes of the resultants are proportional to frequency and increasing with frequency at 6db per octave.

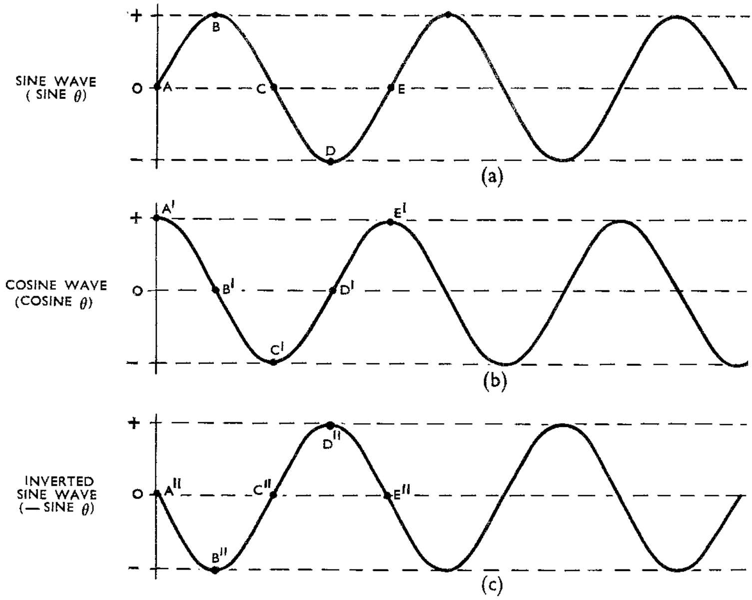


FIG. 25. DIFFERENTIATION OF A SINE WAVE.

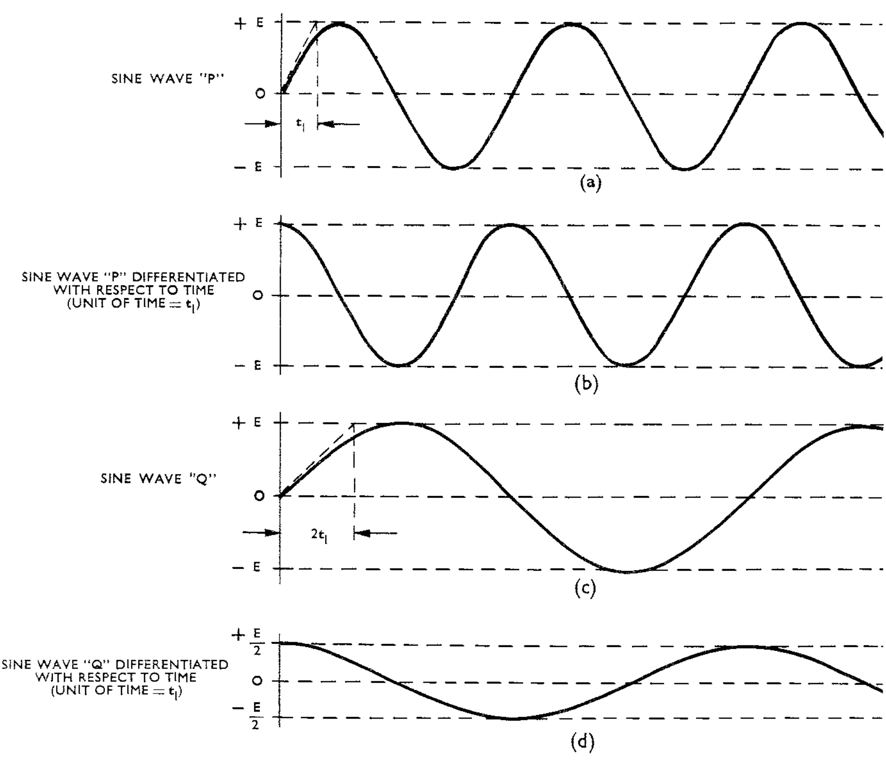


FIG. 26. AMPLITUDE OF DIFFERENTIATED SINE WAVE.

7.5 Differentiation of Complex Waves. The waveforms resulting from differentiation of more complex waves than those already examined can be estimated by considering the rate of change in the same manner as for the preceding examples. In many practical cases the complex waves can be divided into a number of sections similar to the waveforms already considered. Fig. 27 shows some examples.

Differentiation of the "trapezoidal" waveform with linear transitions in Fig. 27a produces a constant positive amplitude during the rise times of the waveform and a constant negative amplitude during the decay times of the waveform. Between the transitions there is no change of amplitude and so the resultant amplitude is zero. The resultant waveform is shown in Fig. 27b.

In Fig. 27c the transitions have a shape that is the same as the peak-to-peak transitions of half of a sine wave; (the transitions are half cycles of a cosine wave). The rate of change of these sections is a maximum in the centre and zero at the extremes. The resultant after differentiation is zero between transitions and increases to a maximum coincident with the centres of the transitions. The resultant is shown in Fig. 27d and is a series of alternatively positive and negative half sine wave shaped pulses coinciding with the transition times.

Fig. 27e is a sawtooth waveform with a practical flyback shape that is one half cycle of a sine wave from the positive peak to the negative peak. During the forward trace, the constant rate of change differentiates to a waveform with a constant positive amplitude. The rate of change of the retrace is zero at the extremes and a maximum negative at the centre, and differentiation of this section produces a waveform that is a negative half cycle of a sine wave. The resultant waveform after differentiation is shown in Fig. 27f.

The example in Fig. 27g when differentiated, gives positive and negative pulses (of infinite amplitude) coinciding with positive and negative transitions respectively and a constant amplitude during the linear sawtooth section of the waveform (Fig. 27h).

As the process of differentiation is related only to rate of change, the presence of a D.C. component in a waveform to be differentiated makes no difference to the resultant.

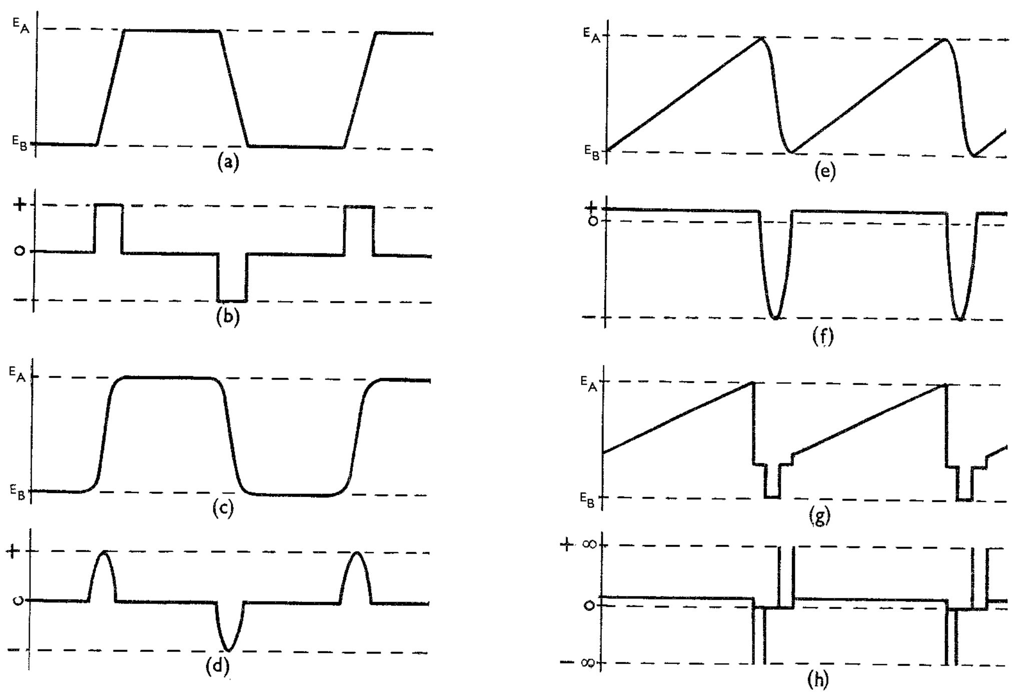


FIG. 27. DIFFERENTIATION OF COMPLEX WAVES.

7.6 Differentiating Circuit. Let us now compare the waveforms of voltage across the components of the R-C circuits of various time constants with the waveforms derived by theoretical differentiation. We see from Fig. 5a that when a square wave is fed into a short time constant circuit, a pulse of voltage is developed across the resistor, and this is coincident with the transitions. Also in Fig. 10a when a sawtooth voltage is fed into a short time constant circuit, the resistor voltage is practically constant for the linear sections of the trace and this voltage is proportional to the rate of change of the input voltage.

A short time constant circuit in which the output is taken across the resistor gives an output waveform similar to the resultant produced by differentiation. Therefore a series capacitor, shunt resistor circuit as in Fig. 28a with a short time constant is a differentiating circuit.

As the voltage across the inductor in a L-R circuit is the same as the voltage across the resistor in an R-C circuit with the same time constant, a series resistor, shunt inductor circuit with a short time constant and with the output voltage taken across the inductor (Fig. 28b) is also a differentiating circuit.

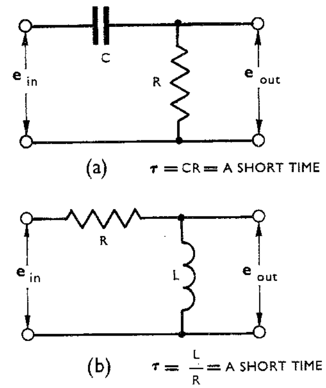


FIG. 28. DIFFERENTIATING CIRCUITS.

The practical waveforms out of a differentiating circuit vary slightly from the ideal waveforms. The pulses of voltage from the R-C differentiating circuit, for a square input, can never be greater than the amplitude of the transitions of the applied voltage, and the pulses have some duration as the voltage decays exponentially towards zero. With a sawtooth voltage input, the output voltage rises exponentially to the maximum and does not step instantly to this value. The shorter the circuit time constant, the closer the shape of the output wave becomes to the ideal. The ideal and practical waveforms are compared in Fig. 29 and these show that an estimation of the output using the principles of calculus gives a good approximation of the actual output. For a rectangular pulse input, the shape of the "practical" output pulse is the same independent of the duration of the input pulse, as long as the circuit time constant is less than 0.2 of the input pulse duration.

The output from a differentiating circuit is independent of any D.C. component that may be present with the input waveform, except immediately following the instant the circuit is first completed, when, due to a switching transient, an additional pulse may be produced, or the initial pulse may alter in amplitude. The results of these switching transients are not normally included in waveform drawings, unless there is some specific reason for doing so.

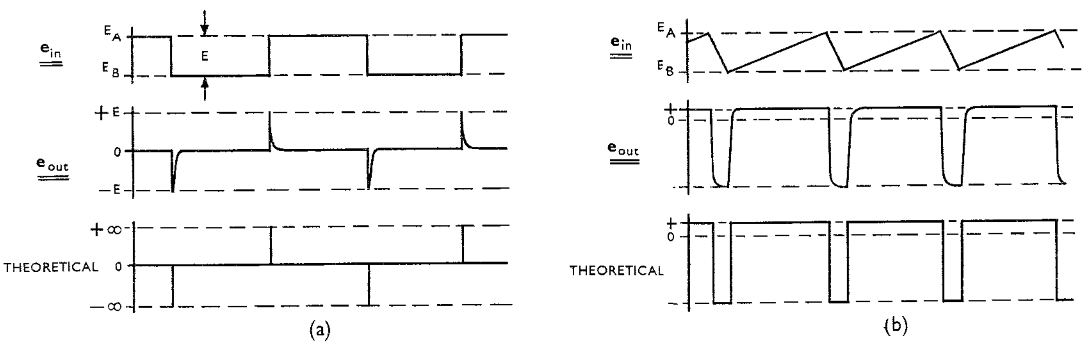


FIG. 29. DIFFERENTIATING CIRCUIT OUTPUT WAVEFORMS.

7.7 Differentiating Circuit with Sine Wave Input. The differentiation of a sine wave is examined in para. 7.4, and the result is a cosine wave, or a leading phase shift of the input wave by 90°. Consider that a 10kc/s sine wave with an R.M.S. amplitude designated E_{in} is fed into a differentiating circuit with a time constant of 2μS and and component values as in Fig. 30a. The voltages across the components in the circuit, and the phase angle between the voltages is calculated as follows:-

$$X_C = \frac{1}{2\pi fC} \quad \left(\frac{1}{2\pi} \simeq 0.16\right)$$

$$= \frac{0.16 \times 10^{12}}{10^4 \times 200}$$

$$= 80{,}000 \text{ ohms.}$$

The R.M.S. value of current (I) in the circuit is common to both C and R,

$$\therefore E_C = I \times 8 \times 10^4 \text{ volts, and}$$

$$E_{out} = E_R = I \times 10^4 \text{ volts.}$$

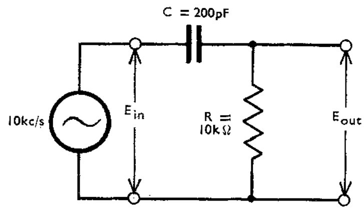
The vector diagram representing the R.M.S. values of voltages in the circuit is shown in Fig. 30b, with E_{in} the vector sum of E_C and E_R.

The phase angle between input and output voltages is given by:-

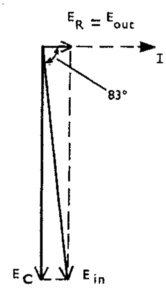
$$\text{Tan}\,\theta = \frac{E_C}{E_R} = \frac{I \times 8 \times 10^4}{I \times 10^4}$$

$$= 8.$$

$$\therefore \theta = 83^\circ \text{ approximately with } E_{out} \text{ leading } E_{in}.$$



(a)



(b)

FIG. 30. DIFFERENTIATING CIRCUIT WITH SINE WAVE INPUT.

The circuit introduces a phase shift approaching the 90° leading phase shift of ideal differentiation. However, a large reduction in amplitude is introduced, and in this example the ratio of the output and input amplitudes is 0.124. The output amplitude corresponds approximately to the theoretical amplitude when the input wave is differentiated with respect to time, where the unit of time is equal to the circuit time constant.

The phase shift introduced by the circuit becomes closer to the ideal when the differentiating circuit time constant is reduced, but the loss at that particular frequency is increased. Similarly, when the input frequency is decreased, a given differentiating circuit introduces a phase shift closer to 90° and the loss increases. With a short time constant compared with the period of one cycle of the input, the output amplitude is, for practical purposes, proportional to frequency, as in the theoretical case.

7.8 Applications of Differentiating Circuits. The pulse shaping performed by the differentiating circuit is used in practical circuits:

- To derive short duration pulses coincident with the transitions of rectangular waves or the flyback of sawtooth waves, for "triggering" the operation of other circuits. Applications are found in television waveform generators, television receiver and monitor synchronizing circuits, waveform monitoring on cathode-ray oscilloscopes, and in counting circuits of frequency measuring equipment, digital voltmeters and digital computers.
- To generate rectangular waves from sawtooth waves as is required to obtain pulses for retrace blanking for waveform monitoring on cathode-ray oscilloscopes.

7.9 Limitations of Practical Differentiating Circuit. There are practical limits which prevent the time constant of a differentiating circuit from being made infinitely short in an attempt to approach the theoretical, and in practice it is not necessary to reduce the circuit time constant to an extreme. The time constant of a circuit often depends on the use to be made of the output, and the information to be extracted from the input. The common application for a differentiating circuit deriving pulses coincident with transitions to trigger other waveform generator circuits, requires pulses to exceed a specified trigger level for a certain time, to achieve reliable triggering. Therefore a very short time constant circuit is not ideal.

When the input wave to be differentiated has a finite rise time, the output voltage reduces as the time constant is reduced, though the output waveform approaches more closely to the shape of the theoretical.

Consider that a "rectangular" wave, with an amplitude of 100V p-p, and, for simplicity, a linear rise time from 0 to 100V of 1µS (Fig. 31a), is fed into a differentiating circuit with a time constant of 1µS. At time A the output voltage from the circuit is zero. During the linear transition, the resistor voltage rises exponentially towards a maximum.

$$E_M = \frac{de_{in}}{dt}\tau = \frac{100 \times 10^6}{10^6} = 100 \text{ volts.}$$

In the transition time of one time constant, e_{out} rises to 0.632 of the maximum; that is to 63.2 volts. During the horizontal section of the wave between B and C, the output voltage decreases exponentially towards zero and reaches this value in 5µS. Coincident with the negative going transitions, the output is of the same shape but is of opposite polarity. The output waveform is illustrated in Fig. 31b.

With a perfect rectangular wave input, the amplitude of the output pulses would have been equal to the amplitude of the transitions, but with the increased rise time, the output pulse amplitude is reduced. There is less reduction of output amplitude when the time constant is increased, but the exponential decay of the pulse becomes longer (Fig. 31c) and usually objectionable. If the desire is for the differentiating circuit to produce trigger pulses coincident with transitions, a time constant comparable with the pulse rise time is a reasonable compromise between output voltage and exponential decay of the output pulse.

Notice that a differentiating circuit with a time constant equal to or greater than the rise time is not short enough to produce an output which approaches the true derivative of the rise time. When the output is to have an amplitude related to the slope of the rise time, the time constant must be short compared to the rise time. However in reducing the time constant the output amplitude is further decreased. The output waveform for a circuit time constant equal to 0.1 of the rise time is shown in Fig. 31d.

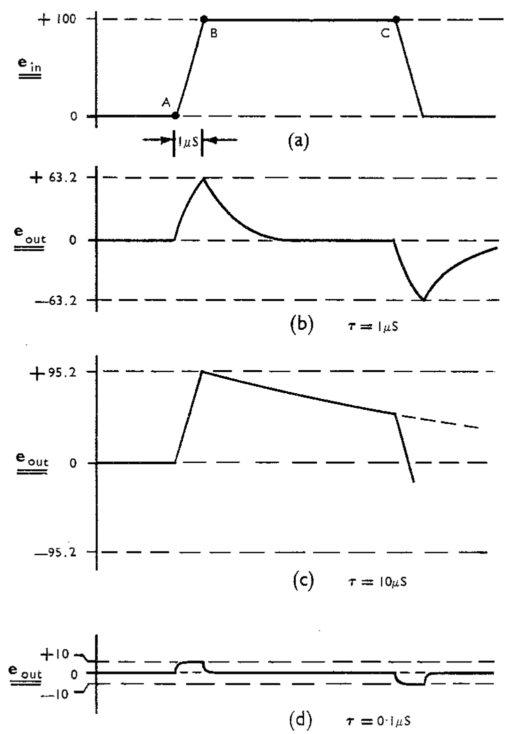


FIG. 31. DIFFERENTIATION OF "RECTANGULAR" WAVES WITH A FINITE RISE TIME.

7.10 Effect of Source Impedance. In the preceding example, the relationship between the circuit time constant and the rectangular wave rise time causes the amplitude of the output pulse to be reduced. The output pulse amplitude is also affected by the relationship between the values of the source and load impedances with respect to the values of the components of the differentiating circuit. If the source impedance is zero and the load impedance is infinite, any two values of capacitance and resistance that gives the desired time constant are satisfactory. Values of source and load impedances, however, set practical limits on the differentiating circuit values.

To examine the effect of a source resistance, consider an ideal square wave (Fig. 32b) is applied to a differentiating circuit through a source resistance as in Fig. 32a. The source resistance must be considered as part of the total resistance when calculating the circuit time constant and voltages existing across the components. The effective time constant is therefore greater than when the differentiating circuit is supplied from a source with zero impedance.

As the circuit time constant is short compared with the period of the square wave, the waveform of voltage across the total circuit resistance is a series of pulses with amplitudes of 200V and with an exponential decay to zero. This voltage is divided between the source resistance of 500Ω and the differentiating circuit resistance of 1500Ω in this example, and gives an output pulse with an amplitude equal to $\frac{1500}{500 + 1500}$, or $\frac{3}{4}$ of the transition amplitude of the source waveform.

The output voltage is a series of pulse with amplitudes of 150 volts, and coincident with the transitions of the square wave as shown in Fig. 32c. The differentiating circuit output waveshape is not changed by the source resistance, but its amplitude is reduced depending on the relative values of the resistance of the differentiating circuit and the total circuit resistance including the source resistance.

The voltage across the resistance of the source is also of the same shape, but it has an amplitude equal to, in this case, $\frac{500}{500 + 1500}$ or $\frac{1}{4}$ of the voltage of the source transition. The input voltage to the differentiating circuit (e_{in}) is therefore no longer a square wave, but is the difference between the source voltage and the voltage across the source resistance. This voltage is shown in Fig. 32d and consists of an initial step of 150 volts followed by an exponential rise to the final voltage. The "rounding" of the transitions of the square wave is the effect of loading of the source by the capacitance of the differentiating circuit.

The requirement for the component values of a differentiating circuit is then, that the resistance of the circuit be much larger than the source resistance. This automatically prevents loading of the source by the circuit capacitance which is determined from the time constant required.

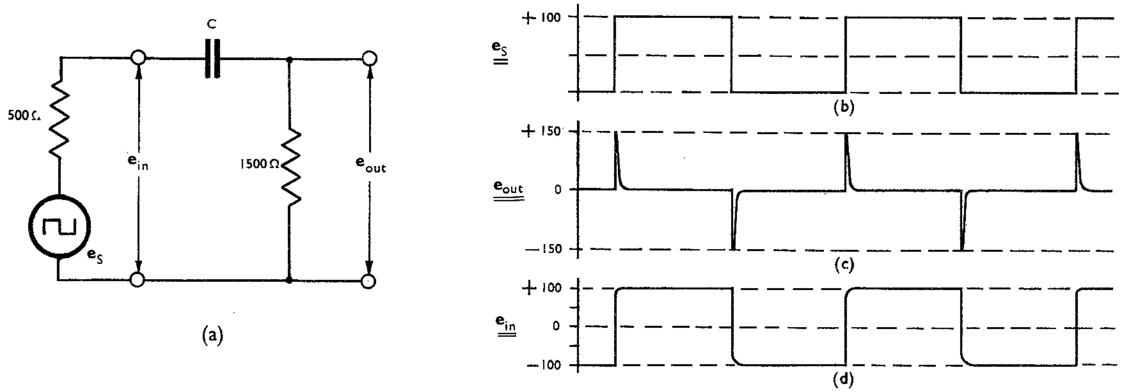


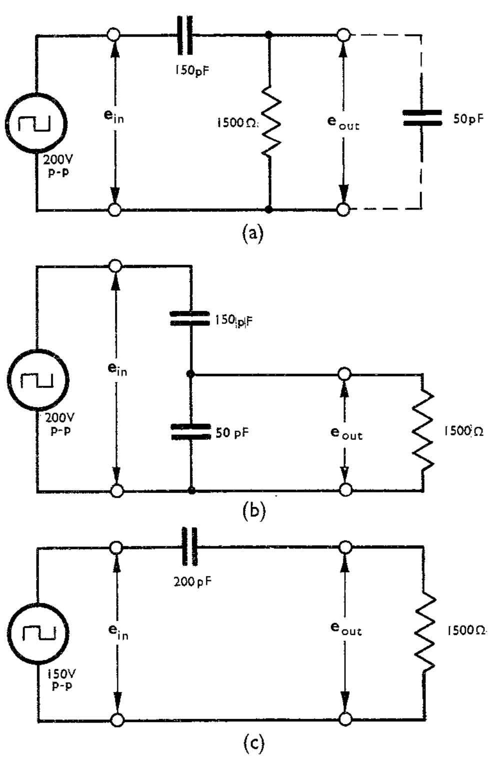
FIG. 32. EFFECT OF SOURCE RESISTANCE.

7.11 Effect of Load Impedance. To examine the effect of the load impedance, consider first the effect of a load resistance. Any resistance across the output is in parallel with the resistance of the differentiating circuit and must be taken into account when calculating the effective time constant of the circuit and the loading of the circuit on the source. A load resistance therefore reduces the effective time constant of the circuit.

A second practical load impedance, often because of wiring, results in capacitance being shunted across the output. The effect of this capacitive loading is examined by considering the circuit shown in Fig. 33a, with a square wave input of 200 volts peak-to-peak from a zero impedance source.

By arranging the circuit as shown in Fig. 33b and considering both capacitors as part of the source, and the resistance as the load, Thevenin's Theorem can be applied to find the output voltage waveform. With the load removed, the source voltage divides across the two capacitors in inverse proportion to their values of capacitance. Therefore, with the 1,500Ω resistor disconnected, e_{out} is a square wave with a voltage of $\frac{150}{50 + 150}$ of e_{in}; i.e. $\frac{3}{4}$ of e_{in}. With the source shorted at e_{in}, the impedance across the output terminals is made up of the two capacitors in parallel, and is 200pF. The equivalent circuit of Fig. 33b includes a generator of 150 volts peak-to-peak supplied via a capacitance of 200pF as in Fig. 33c.

The added capacitance across the output has increased the effective time constant of the circuit, but more important, has reduced the amplitude of the output pulse from the 200 volts pulse without capacitive loading, to 150 volts with capacitive loading. The input and output waveforms for the capacitively loaded differentiating circuit are shown in Fig. 33d. The components of the differentiating circuit must be proportioned, therefore, so that the value of capacitance is large compared with the shunt capacitance across the output.



With a finite impedance in the source providing the input to the differentiating circuit, the two capacitors directly across the input can produce significant loading, and so modify e_{in} and cause a further reduction in the output pulse amplitude.

The two requirements for the values of a practical differentiating circuit, are that the resistance, including any load resistance, be large compared with the source impedance, and that the capacitance be large compared with the load capacitance. These requirements make it harder to provide differentiating circuits, as the required circuit time constant is made shorter.

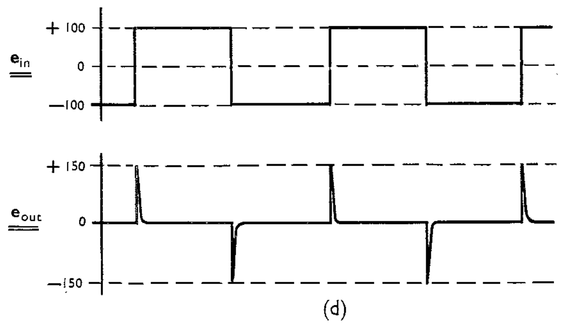


FIG. 33. EFFECT OF CAPACITIVE LOAD ON A DIFFERENTIATING CIRCUIT.

8. INTEGRATION.

8.1 Integration of Waveforms. The principles of calculus are now applied to examine the resultants produced by integrating voltage waveforms with respect to time. Having established integration as the inverse of differentiation, the resultant waveforms produced by integration are summarized by examination of Fig. 34. When a waveform is differentiated, the resultant waveform produced is the one immediately below it. Reading from bottom to top in the figure gives the resultant waveforms produced by integration. Therefore, integrating a series of infinitely short duration pulses of infinite amplitude produces a rectangular waveform which has an infinite rate of change coincident with and in the same direction as the pulses. Between the pulses there is no amplitude and so no change of resultant amplitude. Integrating a rectangular wave produces a sawtooth wave with the rate of change of the sections of the sawtooth dependent on the amplitudes of the sections of the rectangular wave. Integrating a a sawtooth wave where the amplitude is constantly varying results in a parabolic wave where the rate of change of the wave is constantly varying.

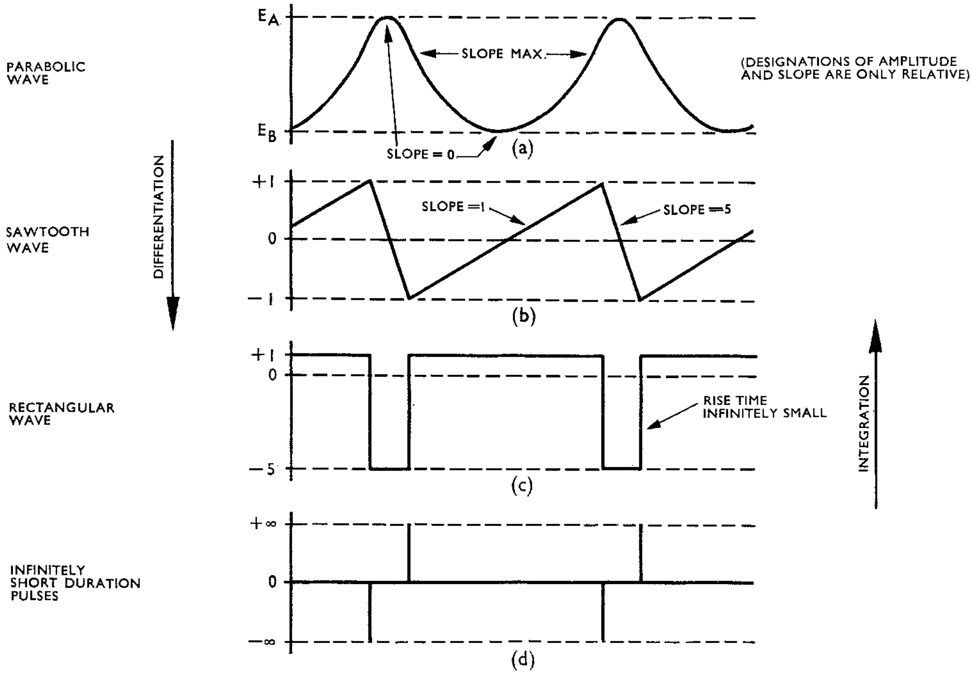


FIG. 34. INTEGRATION OF WAVEFORMS.

8.2 Integration of a Sine Wave. The resultant wave (Fig. 35b) produced by integrating a sine wave can be estimated by examination of Fig. 35a. At point A the sine wave has no amplitude and therefore the resultant at A' has no rate of change. At B, the amplitude is at a positive maximum, therefore at B' the resultant rate of change is also a positive maximum. At C and E the amplitudes are zero therefore the resultant rates of change at C' and E' are also zero. At D the sine wave is at a negative maximum producing a resultant with a maximum negative rate of change at D'.

The resultant wave (Fig. 35b) is still of sine wave shape, but it commences at a negative maximum. This is an inverted cosine wave, and integration of Sine θ gives −Cosine θ. Fig. 35b shows that compared to the input wave in Fig. 35a, integration has introduced a lagging phase shift of 90°.

Integration of the inverted cosine wave produces a further 90° lagging phase shift and the resultant is an inverted sine wave (Fig. 35c). That is, integrating −Cosine θ gives −Sine θ, or when both negative signs are removed, integrating Cosine θ gives Sine θ.

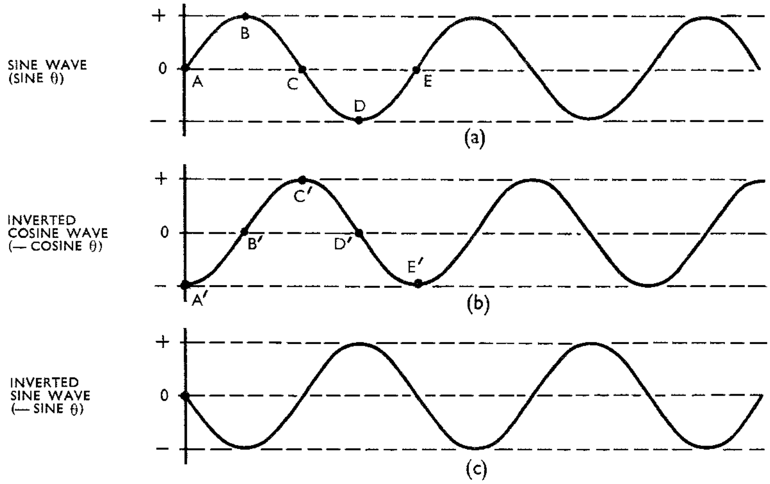


FIG. 35. INTEGRATION OF A SINE WAVE.

When a sine wave is integrated with respect to time and the unit time (t_1) is equal to one radian, the amplitude of the inverted cosine wave produced (Fig. 36b) is equal to the amplitude of the original wave. At any time the resultant is changing at a rate such that it will reach the amplitude of the original signal in the unit of time. Consider that the sine wave in Fig. 36c, with a frequency equal to half that of the sine wave in Fig. 36a, is integrated with respect to time, and the unit is still the time (t_1) for one radian of the original wave. At point B' the rate of change of the resultant is such that it reaches the maximum amplitude of the input (E) in a time t_1. This is twice the rate of change that exists at point A in Fig. 32c, and the resultant wave (Fig. 36d) has twice the amplitude of the original wave. Therefore by integrating sine waves of various frequencies with respect to time, the amplitudes of the resultants are inversely proportional to frequency, and decrease with increase in frequency, at the rate of 6db per octave.

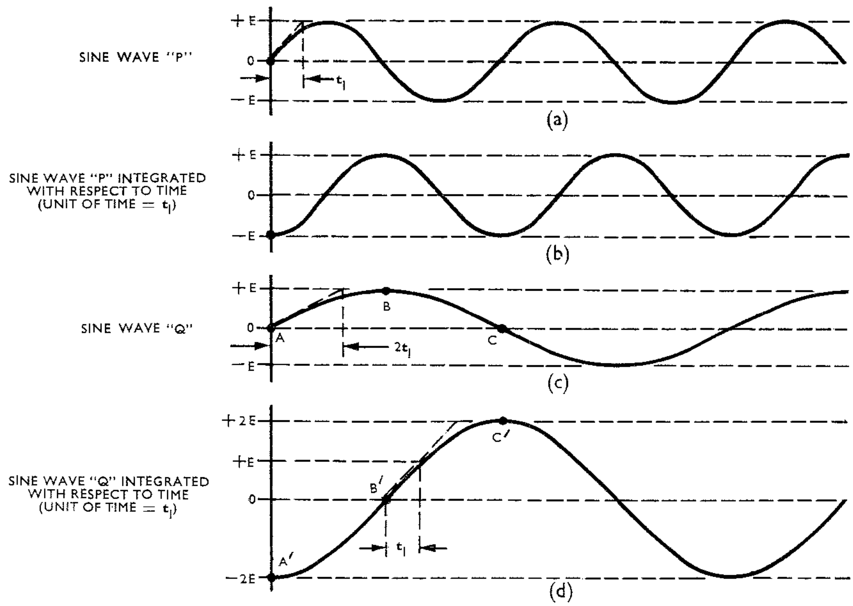


FIG. 36. AMPLITUDES OF INTEGRATED SINE WAVES.

8.3 Integration of Complex Waves. The integration of complex waves follows the inverse of the waveforms considered for differentiation in para. 7.5 and Fig. 27. One further waveform of interest is one containing a D.C. component, for example, the square wave in Fig. 37a. Ignoring the D.C. component, integration of the square wave results in the triangular wave of Fig. 37b. Considering the D.C. component alone, integration of the constant positive amplitude produces a constant increase in the resultant amplitude (Fig. 37c). The addition of the two components in Fig. 37d is the resultant produced by integrating the wave in Fig. 37a. This resultant can be obtained directly. Between A and B the amplitude is large and the resultant (A' to B') increases at a rapid rate. Between B and C the amplitude is small but still positive and the resultant (B' to C') increases but at a slower rate.

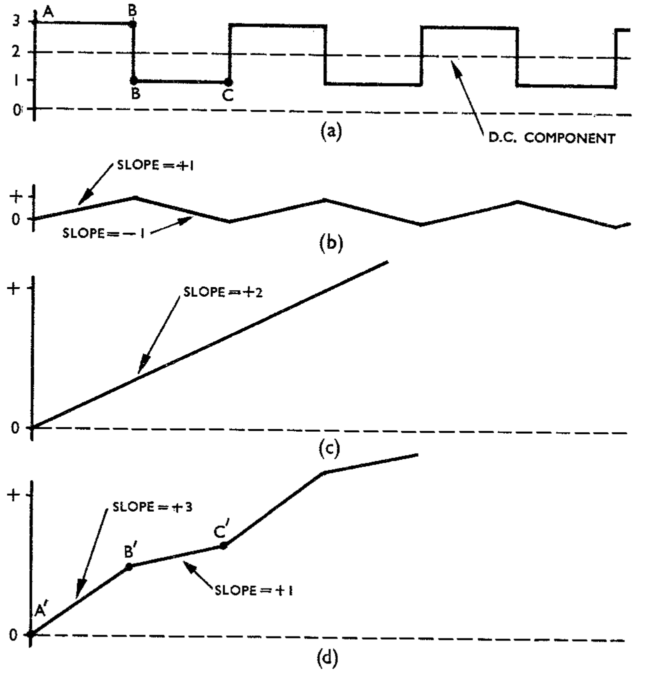


FIG. 37. INTEGRATION OF A D.C. COMPONENT.

8.4 Differentiation and Integration. As integration is the inverse of differentiation, if a wave is differentiated and then the resultant integrated, the original wave should be obtained. This is so, since when the sawtooth wave (Fig. 38a) is differentiated, the resultant is a rectangular wave (Fig. 38b) and by integrating this rectangular wave, the sawtooth wave (Fig. 38c) is obtained again. However, the D.C. component of the signal is lost during differentiation and no information is contained in the rectangular wave to recover this information. Strictly the integration of a rectangular wave produces a sawtooth wave that can have any value of D.C. component which can be included only if some further information of its amplitude is available. This information is often that the resultant starts from zero when integration is commenced, as it is when integration is considered as the summation of areas. The resultants in Fig. 37 show this condition.

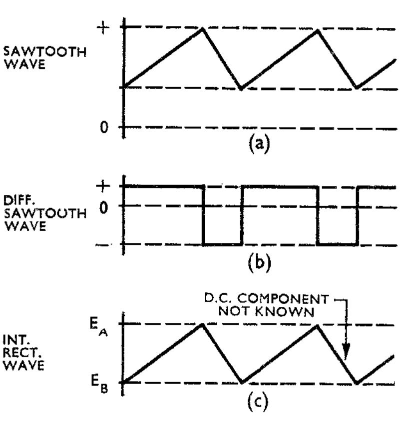


FIG. 38. DIFFERENTIATION AND INTEGRATION.

8.5 Integrating Circuit. Examining the voltages that exist across the components of an R-C circuit we see from Fig. 5b that when a square wave is fed into a long time constant circuit, the voltage across the capacitor is triangular in shape and approximates closely the resultant produced by integration. Also in Fig. 10b with a sawtooth waveform input to a similar circuit, the capacitor voltage is of parabolic shape.

A long time constant circuit with the output taken across the capacitor gives an output waveform similar to the resultant produced by integration. Therefore, a series resistor, shunt capacitor circuit as in Fig. 39a with a long time constant is called an integrating circuit.

Integration is also produced by a series inductor, shunt resistor circuit with a long time constant when the output voltage is taken across the resistor as in Fig. 39b.

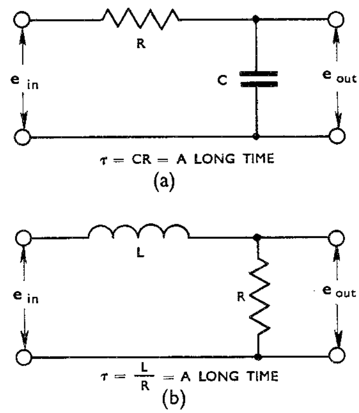


FIG. 39. INTEGRATING CIRCUITS.

The output from a practical integrating circuit cannot equal the ideal resultants produced by integration. When integrating rectangular waves, the output is actually the initial section of an exponential curve, and it varies slightly from the ideal linear sawtooth shape. If, however, the circuit time constant is made long enough, so that the capacitor never charges to more than approximately 10% of the input voltage step, the variation from linearity is practically negligible. This means that the output voltage produced is low, and the circuit introduces considerable loss. The practical integrating circuit deviates from the ideal mainly in that it only integrates the A.C. component of the input signal. The D.C. component of the input signal is transferred to the output. The circuit, however, is able to give some information about changes in the D.C. component of the input voltage.

Consider that the input waveform to the integrating circuit is as shown in Fig. 40. The short duration positive pulses before time t_X have a low value of D.C. component, and after time t_X, the long duration positive pulses have a D.C. component of a much larger value. The output voltage from the circuit before time t_X is a sawtooth wave with extremes on either side of a voltage equal to the D.C. component of the input signal. The average output voltage is low because the charge time of the capacitor is much shorter than the discharge time.

At time t_X, the increase in the pulse duration causes the charge time of the capacitor to be increased at the expense of the discharge time. This allows the average output voltage to increase gradually until it is again varying in a sawtooth manner on either side of the D.C. component of the input signal at this time. The D.C. component of the output signal change approximately exponentially from its initial value to a new value, but includes a superimposed sawtooth signal corresponding to the signal produced by integration of the A.C. component of the input pulses. The output signal from the circuit is shown in Fig. 40.

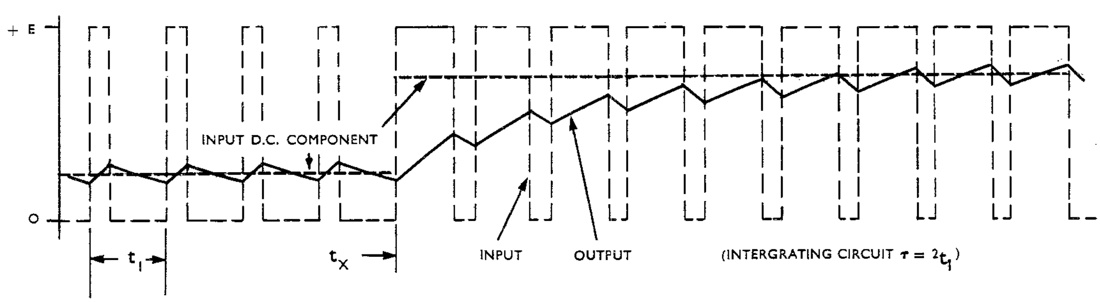


FIG. 40. CHANGE OF D.C. COMPONENT.

8.6 Integrating Circuit with Sine Wave Input. Integration of a sine wave input as examined in para. 8.2 introduces a lagging phase shift of 90°. The following simple example proves that this is approximately true for the integrating circuit. Consider that a 10kc/s sine wave (R.M.S. amplitude - E_{in}) is fed into an integrating circuit with components as shown in Fig. 41a. The circuit time constant is 500µS. The output voltage and the phase shift introduced by the circuit is found as follows:-

$$X_C = \frac{1}{2\pi fC} \qquad (\frac{1}{2\pi} = 0.16)$$

$$= \frac{0.16 \times 10^6}{10^4 \times 0.05} = 320\Omega.$$

The R.M.S. value of current (I) in the circuit is common to both C and R:-

$$\therefore \; E_R = I \times 10^4 \text{ volts, and}$$

$$E_{out} = E_C = I \times 320 \text{ volts.}$$

The vector diagram in Fig. 41b illustrates the R.M.S. values of the circuit voltages with E_{in} the vector sum of E_C and E_R.

The phase angle between input and output voltages is:-

$$\text{Tan } \theta = \frac{E_R}{E_C}$$

$$= \frac{I \times 10^4}{I \times 320} = 31.25.$$

$$\therefore \; \theta = 88^0 \text{ approx. with } E_{out} \text{ lagging } E_{in}.$$

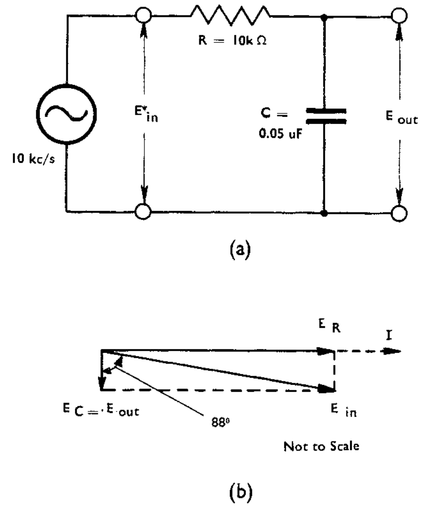


FIG. 41. INTEGRATING CIRCUIT WITH SINE WAVE INPUT.

The practical integrating circuit, then, introduces a phase shift very similar to the 90° lagging phase shift of ideal integration, with the practical circuit becoming closer to the ideal as the time constant is increased. As the input frequency is increased, the time constant becomes longer relative to the time for a cycle of the input, and the phase shift introduced becomes closer to 90°.

The loss introduced by the circuit is large, and in this example the output is 0.032 of the input. When the time constant is long compared with time for one cycle, the output amplitude is inversely proportional to frequency as in the theoretical case, and the output appears to be integrated with respect to time where the unit of time is equal to the circuit time constant.

8.7 Effect of Source and Load Impedance. Because of the long time constant of an integrating circuit, the circuit is relatively immune to effects of source and load impedance. The effect of a source resistance is to increase the circuit time constant, but usually this is desirable in any case. In some practical circuits the source impedance is used as the resistance of the integrating circuit. If it is required to feed another circuit from the same source, loading of the source by the integrating circuit must be prevented. Practically, it is not difficult to achieve this by increasing the value of resistance in the integrating circuit so that it is large by comparison with the source resistance.

Stray capacitance across the output is not difficult to take into account either, especially as the value of shunt capacitance of the integrating circuit is normally much larger than circuit stray capacitance. A resistive load across the output reduces the output voltage and reduces the time constant of the circuit, and if comparable with the integrating circuit resistance, could be objectionable. To prevent this, the integrating circuit can be followed immediately by an amplifier with a high input impedance.

8.8 Applications. Some practical uses for integrating circuits are:

- To segregate pulses of different durations and shapes as is required to identify the vertical synchronizing information in television systems and synchronizing signals in multi-channel pulse modulation systems.
- To generate sawtooth waves from rectangular waves. This is the basis for many sawtooth generators used in cathode-ray oscilloscopes and digital voltmeters.
- To produce special waveforms, related in time to the deflection waveforms of television picture and camera tubes, for the correction of scanning linearity, and changes of brightness and focus occurring in sections of the scanned area.
- To provide the correction for the input signal to a phase modulator so that the output from the modulation is a frequency modulated signal.

Integrating circuits also have applications in analogue computers and are used to make mathematical calculations related to calculus. In this application, and in many cases related to the generation of sawtooth waves, components are included in the feedback circuit of an amplifier so that the circuit as a whole has the same general characteristics as an integrating circuit. Feedback techniques assist in the generation of very accurate sawtooth waveforms for digital voltmeter operation and slightly less accurate sawtooth waveforms for timebases of calibrated C.R.Os. Also, using feedback techniques, circuits can be arranged to give a voltage gain and the output can be much closer to the theoretical result produced by integration. This is required to allow computer calculations with sufficient accuracy.

Differentiating circuits are rarely produced by feedback techniques and rarely found in analogue computers. A differentiating circuit requires the output to increase in proportion to the frequency and this tends to make the amplifier and feedback combination noisy and unstable at high frequencies.

An interesting application for an integrating circuit is in a circuit which allows the magnetization curve of a magnetic material to be displayed on a C.R.O. The magnetization curve shows values of flux density (B) against values of magnetizing force (H). The circuit is arranged as in Fig. 42. For correct operation the circuit values must be chosen so that, at the input frequency, the reactance of the capacitor (C) is small compared with the resistance (R_2), the resistor (R_1) has a small value compared with the reactance (X_L) of the primary inductance (L), and the integrating circuit causes negligible loading of the transformer.

Since R_1 is much less than X_L, R_1 has negligible effect on the circuit current and the horizontal deflection voltage (e_H) across R_1 is proportional to the primary current. But the magnetizing force (H) is proportional to the current, therefore the horizontal deflection is proportional to the magnetizing force.

The voltage across the secondary winding (e_S) is proportional to the rate of change of total flux and therefore e_S is proportional to the derivative of total flux with respect to time. However, the vertical deflection voltage (e_V) is proportional to the integral of e_S with respect to time; therefore e_V is proportional to total flux, and, for a particular magnetic circuit, also to flux density (B). Therefore, the vertical deflection is proportional to flux density. The resulting C.R.O. display shows the B-H curve for the magnetic circuit being examined.

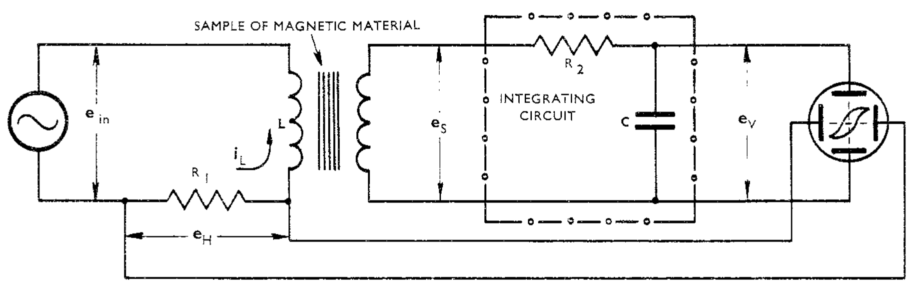


FIG. 42. MAGNETIZATION CURVE DISPLAY USING INTEGRATING CIRCUIT.

9. VOLTAGE DIVIDER.

9.1 Uncompensated Voltage Divider. In electronic circuits, it is often required to reduce the amplitude of a signal, and one simple way of doing this is by means of a resistive voltage divider. Examples of the use of the voltage divider for A.C. signals are the calibrated attenuator in a cathode-ray oscilloscope, and its accessory, the voltage divider probe. The practical voltage divider is complicated by the fact that stray capacitance exists in the circuit, the main capacitance being that across the output. The problem is increased by the requirement that the input impedance of instruments such as oscilloscopes, is required to be high so that it has negligible effect when bridged across the components of the circuit under test.

To examine the properties of the voltage divider, consider that a 9MΩ resistor is added in series with the input circuit of a C.R.O. which has an input impedance represented by a 1MΩ resistor and a 36pF capacitor in parallel. The resultant circuit is as in Fig. 43a, and again in Fig. 43b rearranged and designated for examination.

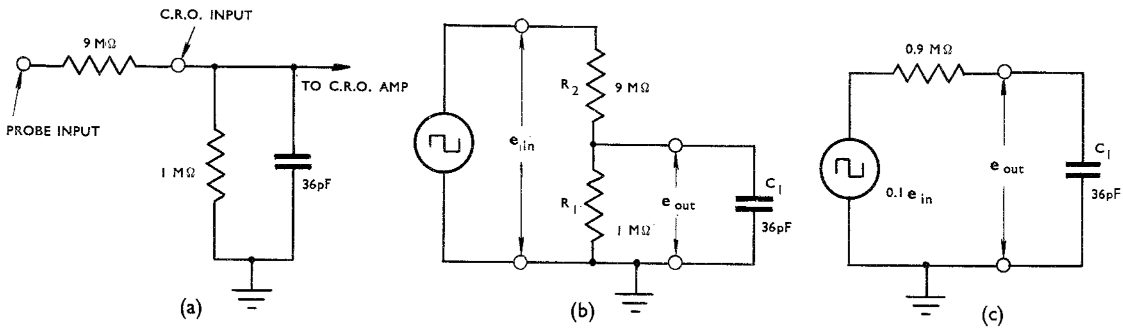


FIG. 43. UNCOMPENSATED VOLTAGE DIVIDER.

Using Thevenin's Theorem, an equivalent circuit is derived as follows:-

The output voltage with the capacitor disconnected is:-

$$= \frac{R_1}{R_1 + R_2} e_{in} \qquad = \frac{1}{1 + 9} e_{in} \qquad = 0.1\ e_{in}.$$

The impedance looking back from the output terminals with the generator replaced by a short circuit is:-

$$= \frac{R_1 R_2}{R_1 + R_2} \qquad = \frac{1 \times 9}{1 + 9} \qquad = 0.9\text{M}\Omega$$

Therefore, the equivalent circuit is as in Fig. 43c.

This equivalent circuit is in the same form as the circuit discussed in Section 5, and will introduce exponential transitions onto a square wave input. The time constant of the circuit is -

$$\tau = CR \qquad = \frac{36 \times 0.9 \times 10^6 \times 10^6}{10^{12}} \qquad = 32.4\mu S.$$

The rise time (t_r) of the output square wave is 2.2 times the circuit time constant, so that the rise time for a waveform through this circuit is:-

$$t_r = 2.2\tau \qquad = 2.2 \times 32.4 \qquad = 71.28\mu S.$$

When it is considered that the waveforms to be examined will typically have rise times of 0.1µS or faster, it is evident that the circuit is of no use. Conditions improve as the resistances of the divider are decreased, but this is no consolation when the circuit is required to have an input impedance as high as possible.

9.2 Compensated Voltage Divider. It is possible to compensate the circuit to offset the effect of the C.R.O. input capacitance by adding the correct value of capacitance across the resistor R_2 as shown in Fig. 44a. Correct compensation is achieved when the time constants $C_1 R_1$ and $C_2 R_2$ are equal. This means that the ratio of the values of the capacitors ($C_1 : C_2$) is the inverse of the ratio of the values of the corresponding resistors ($R_1 : R_2$) and $R_1 : R_2 :: C_2 : C_1$. A mathematical proof is outside the scope of this paper, but the compensation can be justified by considering the resistive and the capacitive arms separately (Fig. 44b).

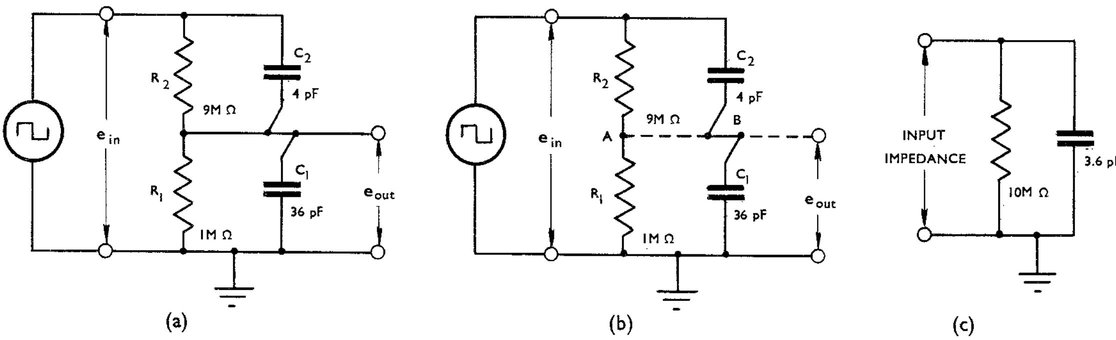


FIG. 44. COMPENSATED VOLTAGE DIVIDER.

The attenuation caused by the resistive section is not frequency conscious and gives an output at point A which is:-

$$= \frac{R_1}{R_1 + R_2} e_{in} = \frac{1}{1 + 9} e_{in} = 0.1\ e_{in}.$$

In the capacitive branch, the voltages divide in inverse proportion to the values of the capacitance, so that the output voltage at point B is:-

$$= \frac{C_2}{C_1 + C_2} e_{in} = \frac{4}{36 \times 4} e_{in} = 0.1\ e_{in}.$$

This output is also independent of frequency, and so no difference of potential exists between A and B at any frequency. Therefore, points A and B can be joined without affecting the circuit and the two branches together give an output which is independent of frequency. This means that the output wave shape is identical with the input, but is reduced in amplitude.

Consider now the resultant input impedance again with the resistive and capacitive arms separated. The impedance is represented by R_1 and R_2 in series, in parallel with C_1 and C_2 in series.

$$R_T = R_1 + R_2 = 1 + 9 = 10M\Omega.$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{4 \times 36}{4 + 36} = 3.6pF.$$

The resultant input impedance is shown in Fig. 44c.

Therefore, by the addition of the compensated voltage divider in the form of the C.R.O. attenuator probe, the advantages gained at the expense of a loss in the ratio of 10 : 1 are, an increase in the input resistance by a factor of 10, and a decrease in the shunt capacitance by a factor of 10, without any alteration of the shape of the wave from input to output. (This calculation has assumed no additional capacitance is introduced by connecting cables.)

9.3 Incorrectly Compensated Voltage Divider. In a practical probe, at least one and possibly both of the capacitors are made slightly variable, and by the adjustment of the trimmer capacitors accurate compensation is achieved while examining the output waveform with a square wave input. It is thus possible to take into account variations of stray capacitance in individual units.

When the value of C_2 is too small, the circuit is under-compensated and an undershoot appears on the output waveform. Coincident with the input transition, the output steps to a voltage dependent on the reciprocal of the values of the capacitors, and then changes exponentially to the final voltage, dependent on the values of the resistors. For small amounts of error in compensation, the exponential change to the final value has a time constant approximately equal to the time constant of the components $(\tau \simeq R_1C_1 \simeq R_2C_2)$.

The output waveform for a 2kc/s square wave input to a slightly under-compensated version of the previous circuit is shown in Fig. 45b. When the circuit is over-compensated by having C_2 too large, an overshoot is produced as in Fig. 45c.

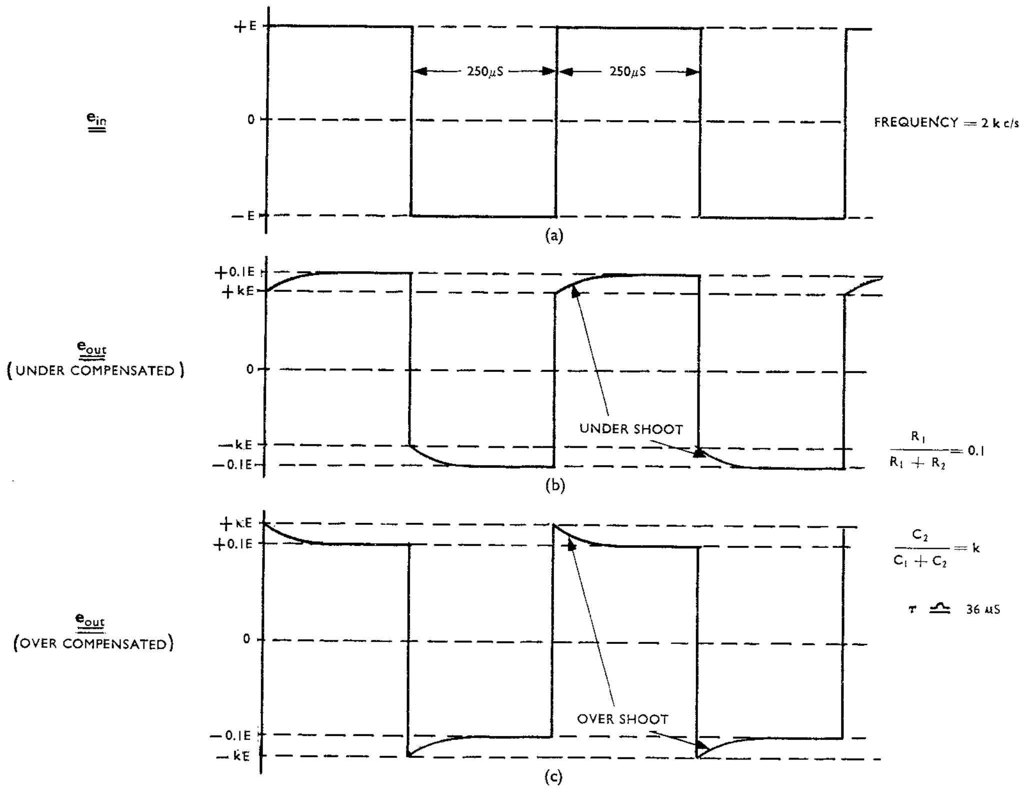


FIG. 45. INCORRECTLY COMPENSATED VOLTAGE DIVIDER.

Note that the time constant of the undershoot and overshoot in this example is relatively long (approximately 36µS) and although there is no theoretical limit to the highest frequency at which compensation occurs, an extremely high frequency square wave, or one with a very short rise time, is not required to make practical adjustments to the compensation capacitor. Actually a high frequency square wave is undesirable. When the exponential change to the final amplitude is not completed in a half cycle of the test square wave, the existance of a small tilt is hard to recognise and adjustment is complicated by changes in amplitude of the displayed waveform.

9.4 Though the example here has been related to the probe of a C.R.O., the same principle of compensation of voltage dividers will be found in many other instruments and circuits.

10. TEST QUESTIONS.

1. A 10kc/s square wave is fed into an R-C circuit consisting of a 100pF capacitor and an 0.1MΩ resistor in series. Illustrate the repetitive waveform expected across each component and calculate the capacitor charging time.

2. Draw a graph to show the voltages present across the components of an R-C circuit, when an input is applied that commences from zero and increases at a constant rate.

3. An R-C circuit with a time constant of 480μS has a sawtooth input voltage with a forward trace time of 43.2mS, a retrace time of 4.8mS, and an amplitude of 36 volts peak-to-peak.

   (i) Draw the waveform of voltage across the resistor. (ii) Calculate the peak voltages.

4. By inspection, differentiate the waveforms in Fig. 46. Show relative amplitudes for the sections of each resultant.

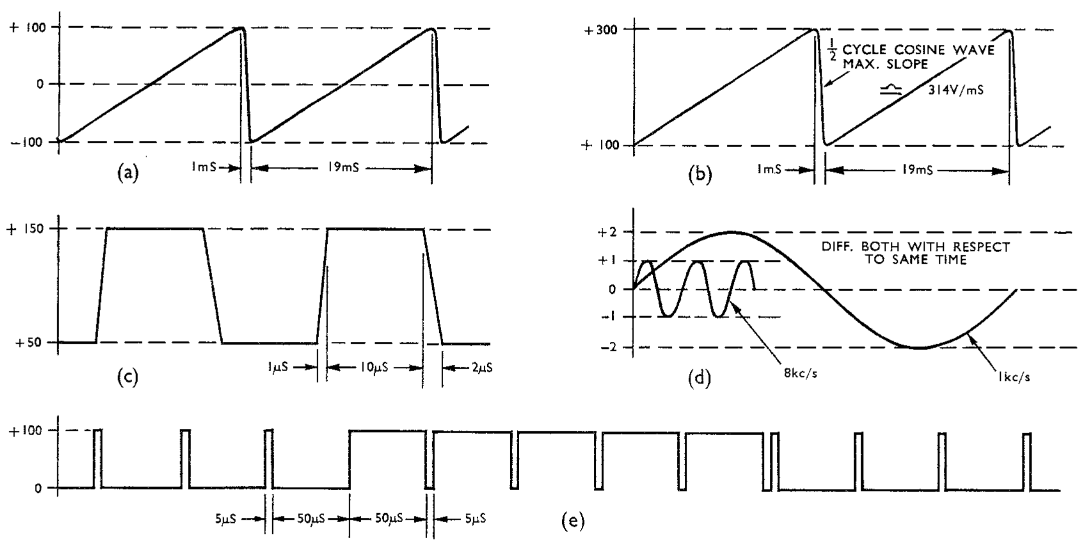


FIG. 46. WAVEFORMS FOR QUESTION 4.

5. What phase shift is introduced on to a sine wave by a differentiating circuit?

6. A sawtooth current waveform passes through a resistor and an inductor in series. Using the information that a sine wave voltage across the inductor leads the sine wave current through it by 90°, i.e. the inductor voltage is proportional to the derivative of the inductor current with respect to time, estimate the shape of the voltage across the combined resistance and inductance.

7. A 16kc/s square wave is fed into the grid-cathode circuit of a valve which has an anode resistance (r_a) of 10kΩ and an inductive anode load of 10mH. Assuming that the valve operates on a linear part of the characteristic, and neglecting any circuit capacitance, what is the output waveform? (Draw the equivalent of the valve circuit.)

8. How is the operation of a differentiating circuit affected by (i) Output shunt capacitance, (ii) Source resistance.

9. A differentiating circuit is to be fed from a 10kΩ source, is to have a time constant of 20μS, and a 20pF shunt capacitance is expected across the output. Which of the following pairs of components would be the most suitable?

   (i) 20pF and 1MΩ, (ii) 200pF and 100kΩ,
   (iii) 2000pF and 10kΩ, (iv) 0.02μF and 1kΩ.

10. Draw the output waveforms from an R-C differentiating circuit when the input waveform is as in Fig. 46e (τ = 0.5μS).

11. A square wave of 50c/s is fed into the circuit in Fig. 47. Estimate the shape of the output voltage and determine approximately the amplitudes at significant points.

12. Illustrate to show the output waveform when the waveform in Fig. 46e is fed into an integrating circuit with a time constant of:- (i) 25μS, (ii) 125μS.

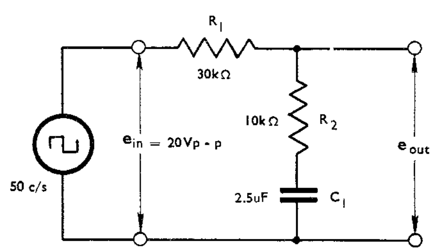


FIG. 47. CIRCUIT FOR QUESTION 11.

13. A coupling circuit at the input of a C.R.O. consists of an 0.25μF capacitor and an 0.5MΩ resistor. The C.R.O. is used to examine the waveform on the anode of a valve where the D.C. voltage as measured with a moving coil multimeter is +200V. The waveform present on the anode is a 20c/s square wave of 120 volts peak-to-peak.

   (i) Assuming the capacitor to be initially without charge, how long would it take for the C.R.O. display to establish a steady vertical position?

   (ii) What tilt is introduced on to the waveform by the coupling circuit?

   (iii) In this circuit, at what frequency would the reactance of the capacitor equal the circuit resistance?

   (iv) Draw approximately the waveform displayed on the C.R.O., and include the voltages present at the extremes of each transition as measured by the C.R.O. Assume that the input coupling circuit causes the only waveform degradation.

14. By inspection, integrate the waveforms in Fig. 48 within the limits shown. Indicate relative slopes of the output waveform where possible.

15. A square wave is fed into the grid-cathode circuit of an uncompensated valve amplifier. What is the rise time of the voltage present on the anode of the valve? The characteristics of the amplifier circuit are:-

   (i) Valve anode resistance (r_a) = 10kΩ,

   (ii) Anode load resistance = 2kΩ,

   (iii) Total capacitance between anode and cathode = 30pF.

   (Draw the equivalent circuit of the amplifier stage.)

16. One stage of a multivibrator is direct coupled to its associated stage as in Fig. 49. Positive rectangular pulses of 1,000c/s and with a pulse duty factor of 0.2 are present on the anode of V_1. The total shunt capacitance in the grid cathode circuit of V_2 is 20pF.

   (i) Draw the waveform expected on the grid of V_1.

   (ii) What is the rise time of this waveform?

   (iii) Where can a component be added to improve this waveform?

   (iv) What value of component is required for "ideal" compensation?

17. An audio amplifier transformer has a primary inductance of 15H and a load resistance of 5,000Ω reflected into the primary from the secondary. What is the tilt produced on a 400c/s square wave when the transformer is used with a pentode valve having an anode resistance (r_a) of 45kΩ. The equivalent circuit at low frequencies is as shown in Fig. 50.

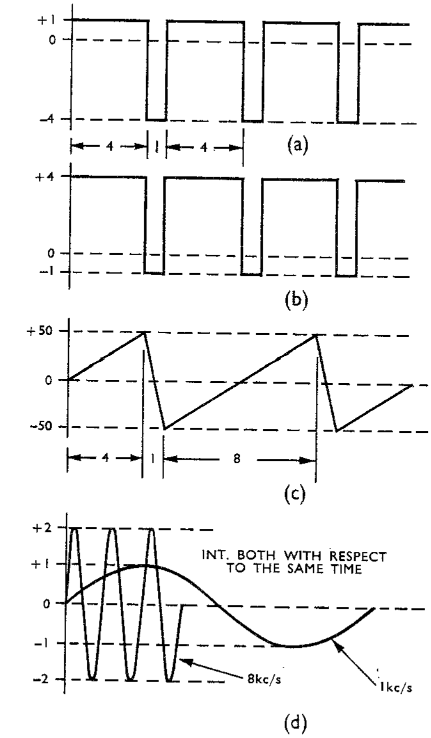


FIG. 48. WAVEFORMS FOR QUESTION 14.

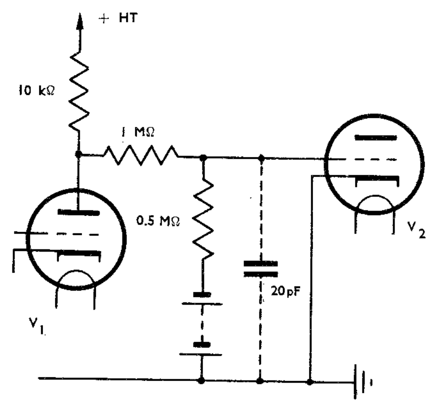


FIG. 49. CIRCUIT FOR QUESTION 16.

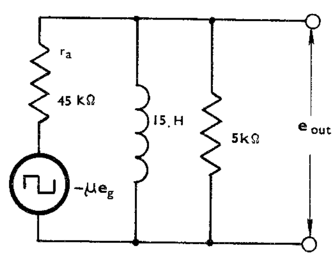


FIG. 50. CIRCUIT FOR QUESTION 17.

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