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COURSE OF TECHNICAL INSTRUCTION

Engineering Training Section, Headquarters, Postmaster-General's Department, Melbourne C.2.

THE SMITH CHART FOR R.F. TRANSMISSION LINES

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1. INTRODUCTION.

1.1 The properties of an R.F. transmission line can be presented graphically in a variety of charts of which the most widely used is the Transmission Line Calculator or Smith Chart (see Fig. 1 on page 2). This chart is useful in H.F. and V.H.F. work, because it eliminates the need for complex mathematical calculations in solving most transmission line problems. It is also used at U.H.F. where electrical measurements must be made indirectly.

1.2 The Smith Chart provides a means of plotting impedance conditions along a loss-free transmission line so that, given the conditions at one point, the conditions at any other point along the line can be calculated easily and quickly. For example, when a transmission line is not terminated in its characteristic impedance, standing waves result and the input impedance of the line depends on its length. When the terminatin impedance is known, the Chart can be used to determine the input impedance of the line for any length. Conversely, with a given line length and a known (or measured) input impedance, the load impedance may be determined. The Chart applies specifically to a loss-free line but it is possible to make allowance for attenuation; often, in practice, attenuation can be neglected.

1.3 This paper gives a basic idea of the Smith Chart and discusses simple applications of the scales provided on most versions of the Chart.

2. IMPEDANCE CO-ORDINATE SYSTEM.

2.1 The use of the Smith Chart (Fig. 1) is similar to the use of a graph. The Chart can be considered as a specialized type of graph with curved, rather than rectangular, co-ordinate lines. The co-ordinate system consists basically of two groups of circles, one group for resistance and the other group for reactance.

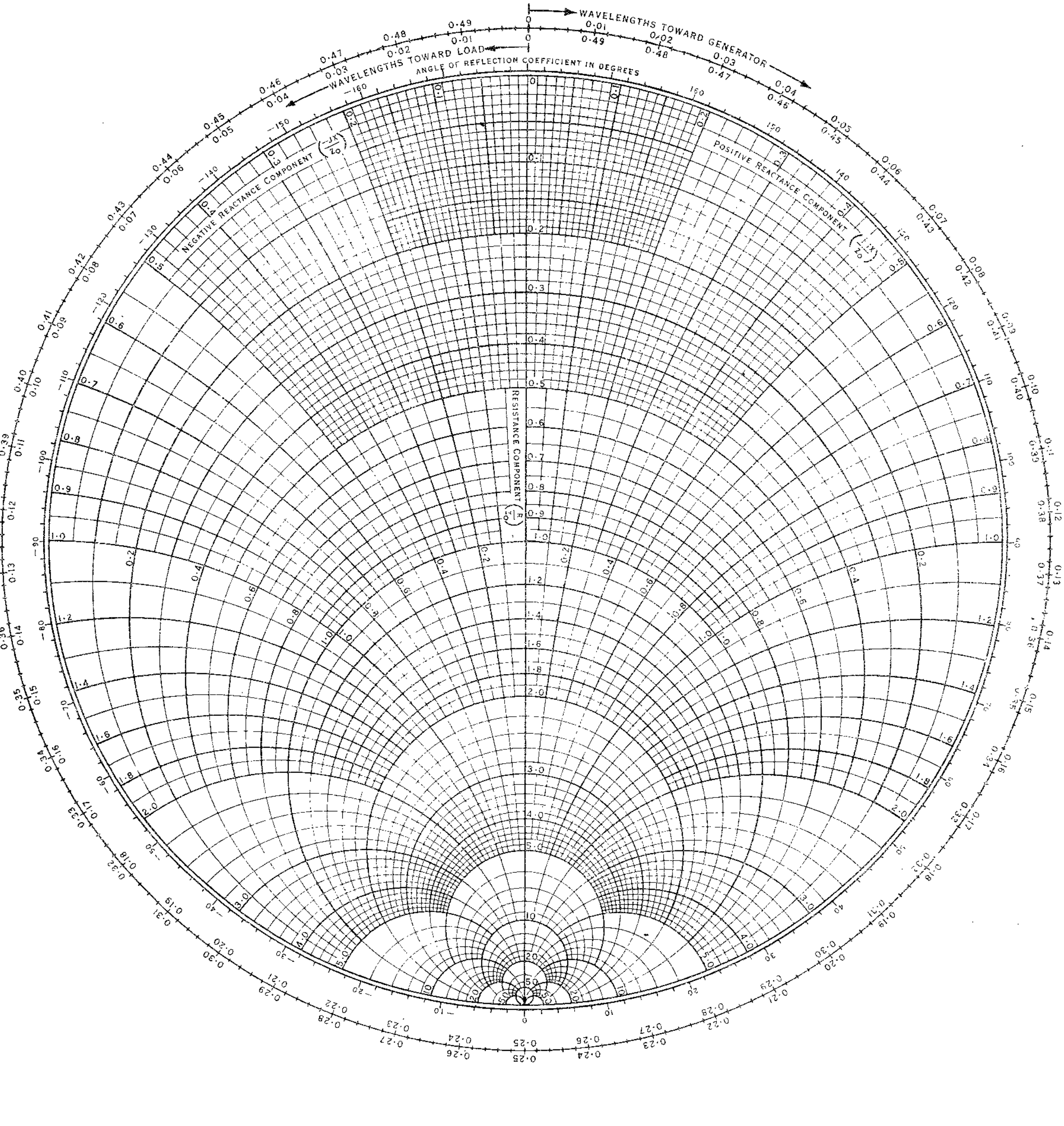


FIG. 1. THE SMITH CHART.

2.2 The Resistance Circles (Fig. 2) are centred on the resistance axis (the only straight line on the Chart), and have a common tangent with the outer circle at the bottom of the Chart.

Each circle is assigned a value of resistance and all points along any one circle have the same resistance value. The values assigned to these circles vary from zero at the top of the chart to infinity at the bottom, and represent a ratio with respect to the value assigned to the centre point of the Chart, indicated 1.0. This centre point is called prime centre.

When prime centre is assigned a value of 100 ohms, then 200 ohms resistance is represented by the 2.0 circle, 50 ohms by the 0.5 circle, 20 ohms by the 0.2 circle, and so on.

When a value of 50 ohms is assigned to prime centre, the 2.0 circle now represents 100 ohms, the 0.5 circle 25 ohms, and the 0.2 circle 10 ohms.

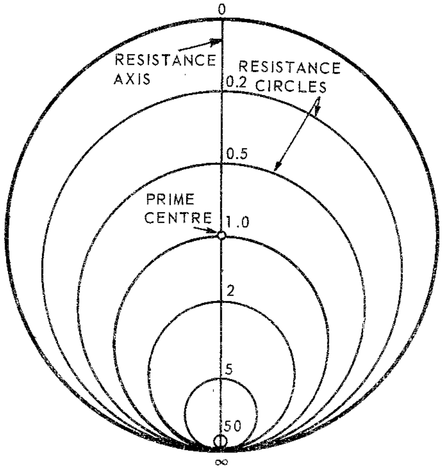


FIG. 2. RESISTANCE CIRCLES.

In each case, the value to be plotted on the Chart is determined by dividing the actual resistance by the number assigned to prime centre. The process is called normalizing.

When solving transmission lines problems, it is customary to assign to prime centre a value equal to the Zo of the line being used and to record this value at the start of the calculations to avoid possible confusion later. Remember that the Zo of a transmission line is regarded as being purely resistive at radio frequencies.

Conversely, values from the Chart are converted back to actual resistance values by multiplying the Chart value by the value assigned to prime centre. This feature permits the Smith Chart to be used for any impedance value, and therefore with any type of uniform transmission line, whatever its Zo may be.

Specialized versions of the Smith Chart are available with a value of 50, 75 or 600 at prime centre. These are intended for use with 50 ohm, 75 ohm or 600 ohm lines respectively.

2.3 The Reactance Circles (Fig. 3) appear as curved lines on the Chart because only segments of the complete circles are drawn. These circles are tangent to the resistance axis, which itself is a member of the reactance family (with a radius of infinity). The centres are displaced to the right or left on a line tangent to the bottom of the chart. The large outer circle bounding the co-ordinate portion of the Chart is the reactance axis.

Each reactance circle segment is assigned a value of reactance and all points along any one segment have the same reactance value. As with the resistance circles, the values assigned to each reactance segment are normalized with respect to the value assigned to prime centre. Values to the right of the resistance axis are positive (inductive), and those to the left of the resistance axis are negative (capacitive).

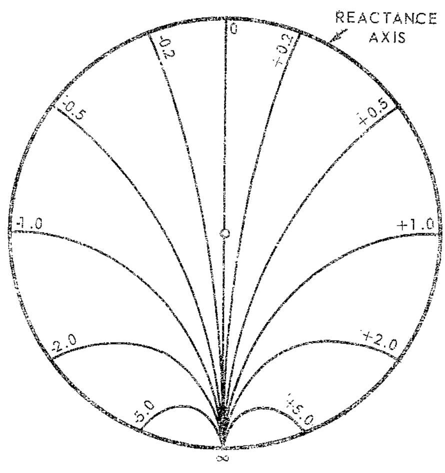


FIG. 3. REACTANCE CURVES.

3. PLOTTING RESISTANCE, REACTANCE AND IMPEDANCE.

3.1 The resistance and reactance curves are combined to form the co-ordinate system of the Smith Chart. Pure resistance, pure reactance or complex series impedance can be plotted on this co-ordinate system, as shown in the following examples. From these examples, note that the same resistance, reactance or impedance can be plotted at different points on the Chart, depending on the value assigned to prime centre.

3.2 A pure resistance is plotted along the vertical resistance axis at a point equal to the ratio of the resistance to the value assigned to prime centre. For example -

(i) When prime centre = 50 ohms, a resistance of 200 ohms (Z = 200 + j0) is plotted at point 4.0 on the vertical resistance axis; and a resistance of 30 ohms (Z = 30 + j0) at point 0.6.

(ii) When prime centre = 100 ohms, a resistance of 200 ohms is plotted at point 2.0; and a resistance of 30 ohms at point 0.3.

3.3 A pure reactance is plotted around the circular reactance axis at a point equal to the ratio of the reactance to the value assigned to prime centre. Inductive reactances are plotted on the right-hand (positive) half of the reactance axis, and capacitive reactances on the left-hand (negative) half. For example -

(i) When prime centre = 50 ohms, an inductive reactance of 200 ohms (Z = 0 + j200) is plotted at point 4.0 on the positive half of the circular reactance axis.

(ii) When prime centre = 100 ohms, a capacitive reactance of 200 ohms (Z = 0 - j200) is plotted at point 2.0 on the negative half of the circular reactance axis.

3.4 Impedances are plotted at the points of intersection of the resistance and reactance curves. Impedances which are predominantly inductive are plotted on the positive (right-hand) side of the resistance axis; impedances which are predominantly capacitive are plotted on the negative (left-hand) side. For example -

(i) Consider an impedance consisting of 50 ohms resistance in series with 100 ohms inductive reactance (Z = 50 + j100). When prime centre = 100 ohms, we normalize this impedance by dividing each component by 100. The normalized impedance is -

$$\frac{50}{100} + j\frac{100}{100} = 0.5 + j1.0.$$

This impedance is plotted at the point of intersection of the 0.5 resistance circle and the +1.0 reactance circle. (Point A in Fig. 4).

(ii) When prime centre = 50 ohms, the same impedance (50 + j100) is plotted at the intersection of the 1.0 resistance circle and the +2.0 reactance circle, that is, at 1 + j2. (Point B in Fig. 4).

(iii) With a resistance of 300 ohms in series with 750 ohms capacitive reactance (assuming prime centre = 600 ohms), the impedance of 300 - j750 ohms is plotted at 0.5 - j1.25. (Point C in Fig. 4).

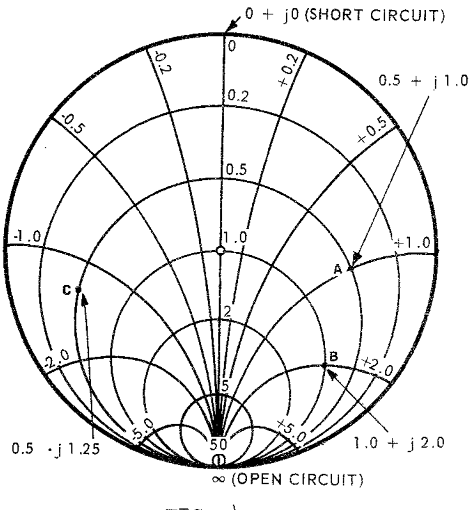


FIG. 4.
RESISTANCE AND REACTANCE CURVES.

3.5 Short and Open Circuits. A true short circuit has zero resistance and zero reactance, or 0 + j0. This impedance is plotted at the top of the Chart, at the intersection of the resistance and reactance axes. An open circuit has infinite resistance, and is plotted at the bottom of the Chart, at the intersection of the resistance and reactance axes.

EXERCISES

1. *On the Smith Chart shown in Fig. 5, plot and label the following impedance values:-*

Point	Impedance (ohms)	Prime Centre (ohms)	Point	Impedance (ohms)	Prime Centre (ohms)
A	40 + j0	100	E	42.5 + j22.5	50
B	200 - j50	100	F	60 - j40	75
C	200 - j50	50	G	0 + j300	600
D	75 + j250	50	H	300 $\angle 0^{\circ}$	600

2. *In Fig. 5, state the impedance (R ± jX) indicated by:-*

   (a) Point I (prime centre = 100 ohms).
   (b) Point J (prime centre = 200 ohms).
   (c) Point K (prime centre = 600 ohms).
   (d) Point L (prime centre = 600 ohms).

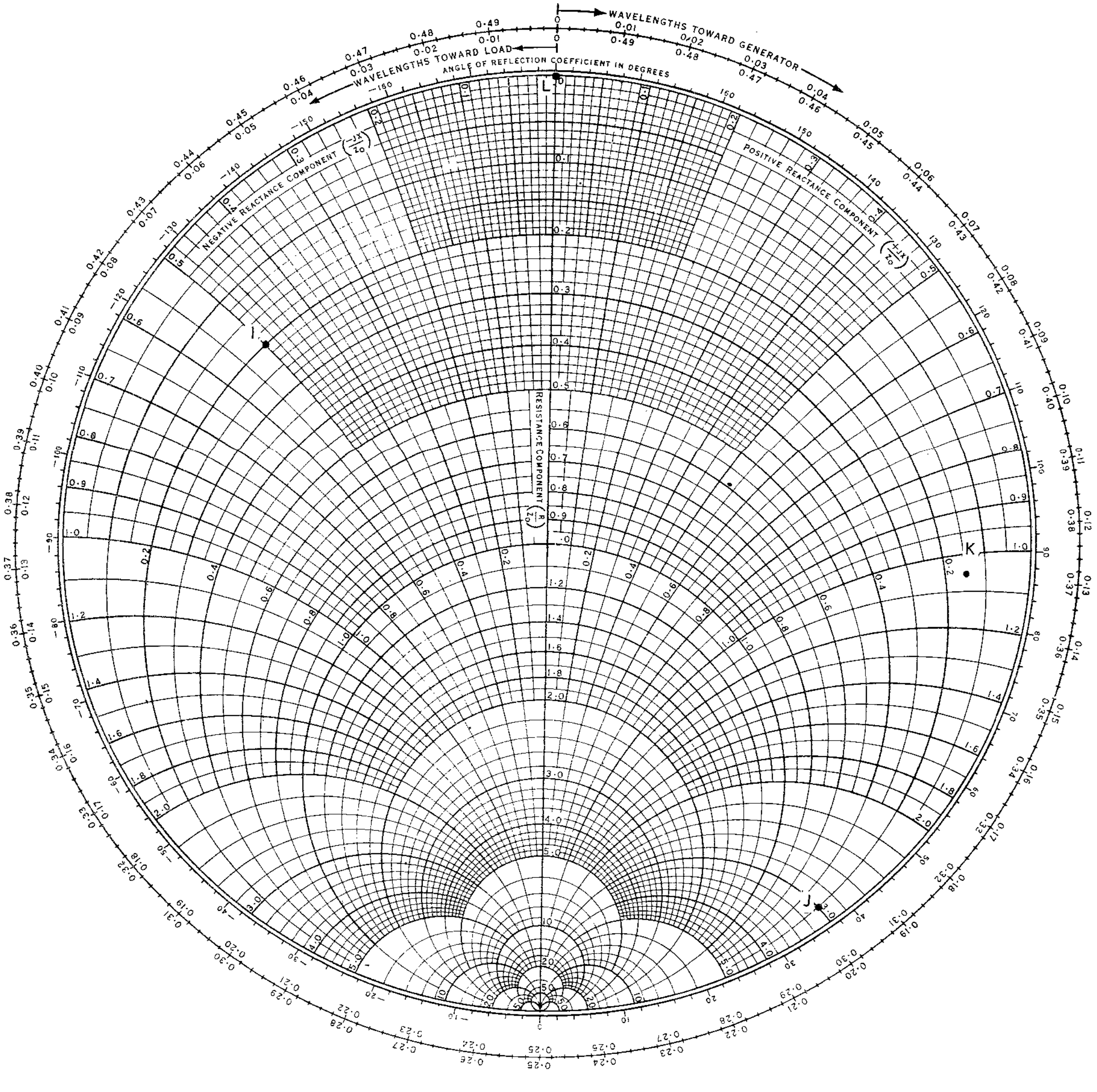


FIG. 5. PLOTTING IMPEDANCES ON THE SMITH CHART.

4. WAVELENGTH SCALES.

4.1 The Wavelength Scales near the outer perimeter of the Chart, are calibrated in fractions of an electrical wavelength along a transmission line. The clockwise scale starts at the load and progresses toward the generator or input end of the line. The anti-clockwise scale starts at the generator and proceeds toward the load. The complete circle represents one half wavelength.

Progressing once around the perimeter of these scales corresponds to progressing along a transmission line for a half wavelength. Because impedances repeat every half wavelength along a line, the Chart can be used for any length by subtracting an integral, or whole number of half wavelengths from the line's total length.

4.2 This Chart can be used to determine the input reactance of a transmission line of known electrical length, which is terminated in either a short circuit, an open circuit, or a pure reactance. The procedure is as follows:-

(i) On the reactance axis, plot the value of line termination, normalized with respect to the Zo of the line.

(ii) Draw a straight line (or lay a straight edge) from prime centre through the plotted point in (i) to intersect the "Wavelengths toward Generator" scale, and read the value from the scale.

(iii) To the reading in (ii), add the electrical length of the line and plot this value on the clockwise wavelength scale.

(iv) Draw a straight line from prime centre to the wavelength reading in (iii) and read the normalized value from the reactance axis at the point of intersection.

(v) Multiply the normalized value in (iv) by the Zo of the line to find the line input reactance.

4.3 Conversely, the Chart can be used to determine the electrical lengths which will give a particular value of input reactance, of a transmission line terminated in either a short circuit, an open circuit or a pure reactance. The procedure is as follows:-

(i) On the reactance axis, plot the values of line termination and input reactance normalized with respect to the Zo of the line.

(ii) Draw straight lines from prime centre through the plotted points in (i) to intersect the "Wavelengths towards Generator" scale, and read the values from this scale.

(iii) Subtract the wavelength readings in (ii) to find the shortest electrical length of transmission line. (Other lengths are determined by adding multiples of 0.5λ to the shortest length).

4.4 Short Circuited Lines. Fig. 6 shows how to use the Chart to find the input reactance of a 600 ohm line 0.125λ long, which is terminated in a short circuit. Using the steps listed in para. 4.2 -

(i) Plot the impedance of the short circuit termination (0 + j0) at the top of the chart.

(ii) Draw a straight line from prime centre through this plotted point, to intersect the "Wavelengths towards Generator" scale at 0λ.

(iii) From 0λ, proceed a distance of 0.125λ around the clockwise wavelength scale.

(iv) Draw a straight line from prime centre to the 0.125λ point. Note that this line intersects the reactance axis at 0 + j1.0.

(v) Multiply this normalized value by 600 ohms to get 0 + j600 ohms. The input reactance of this line is, therefore, 600 ohms, and it is purely inductive.

Similarly, when this short circuited line has a length of 0.31λ, the normalized value of input reactance is 0 - j2.55. Multiplying this value by 600 ohms, we get 0 - j1,530 ohms. The input reactance of this line is 1,530 ohms, and it is purely capacitive.

When the length of a short circuited line is 0.25λ, its Zin is, theoretically, infinite (that is, an open circuit). A line which is an odd number of quarter wavelengths long, inverts the load.

When the length of a short circuited line is 0.5λ, its Zin is, theoretically, zero (that is, a short circuit). A line which is an even number of quarter wavelengths long, repeats the load.

For lengths greater than 0.5λ, subtract a whole number of half wavelengths from the length to give a value between 0λ and 0.5λ. Thus, for 0.625λ, subtract 0.5λ and read the value of input reactance as for 0.125λ.

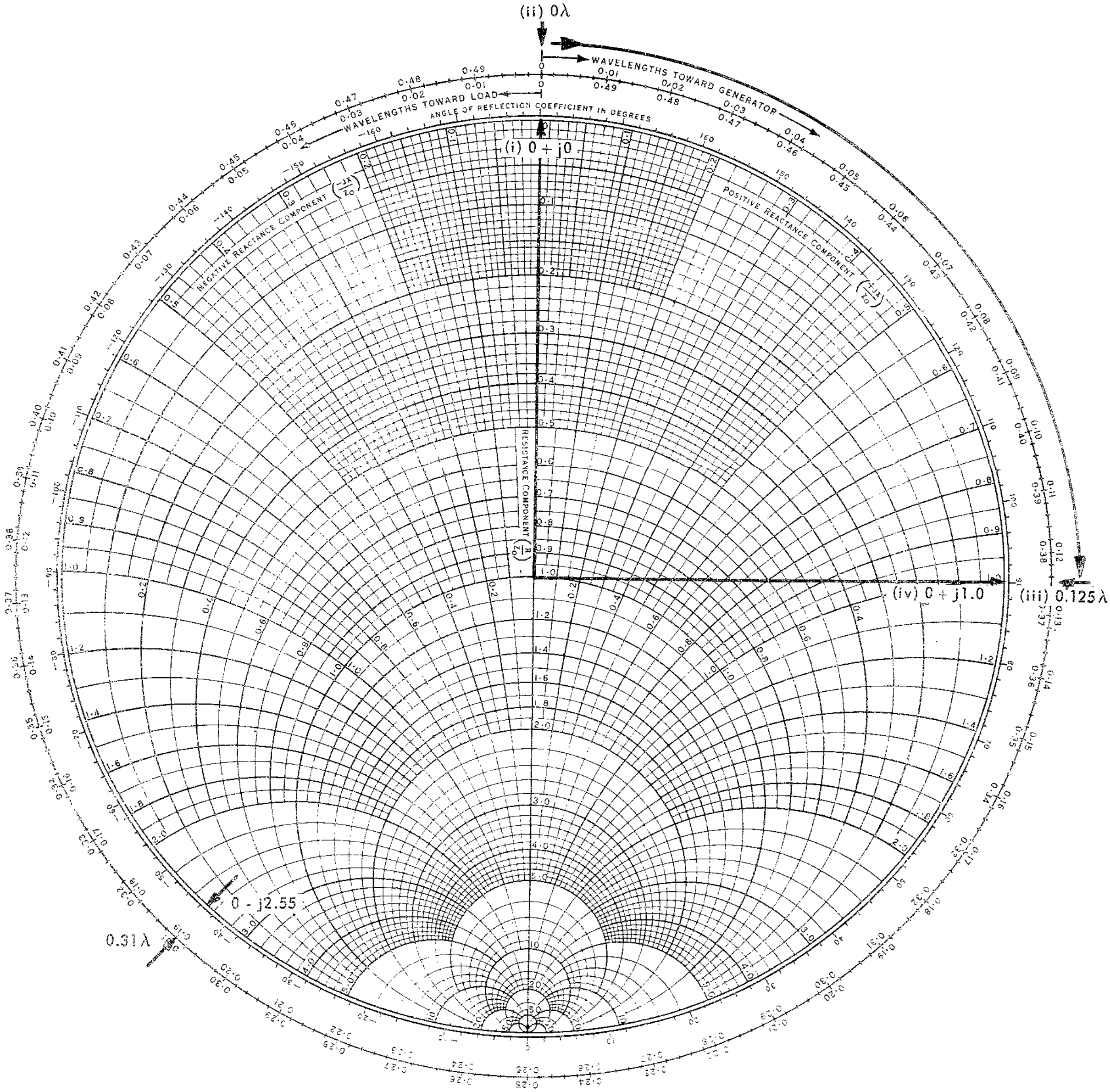


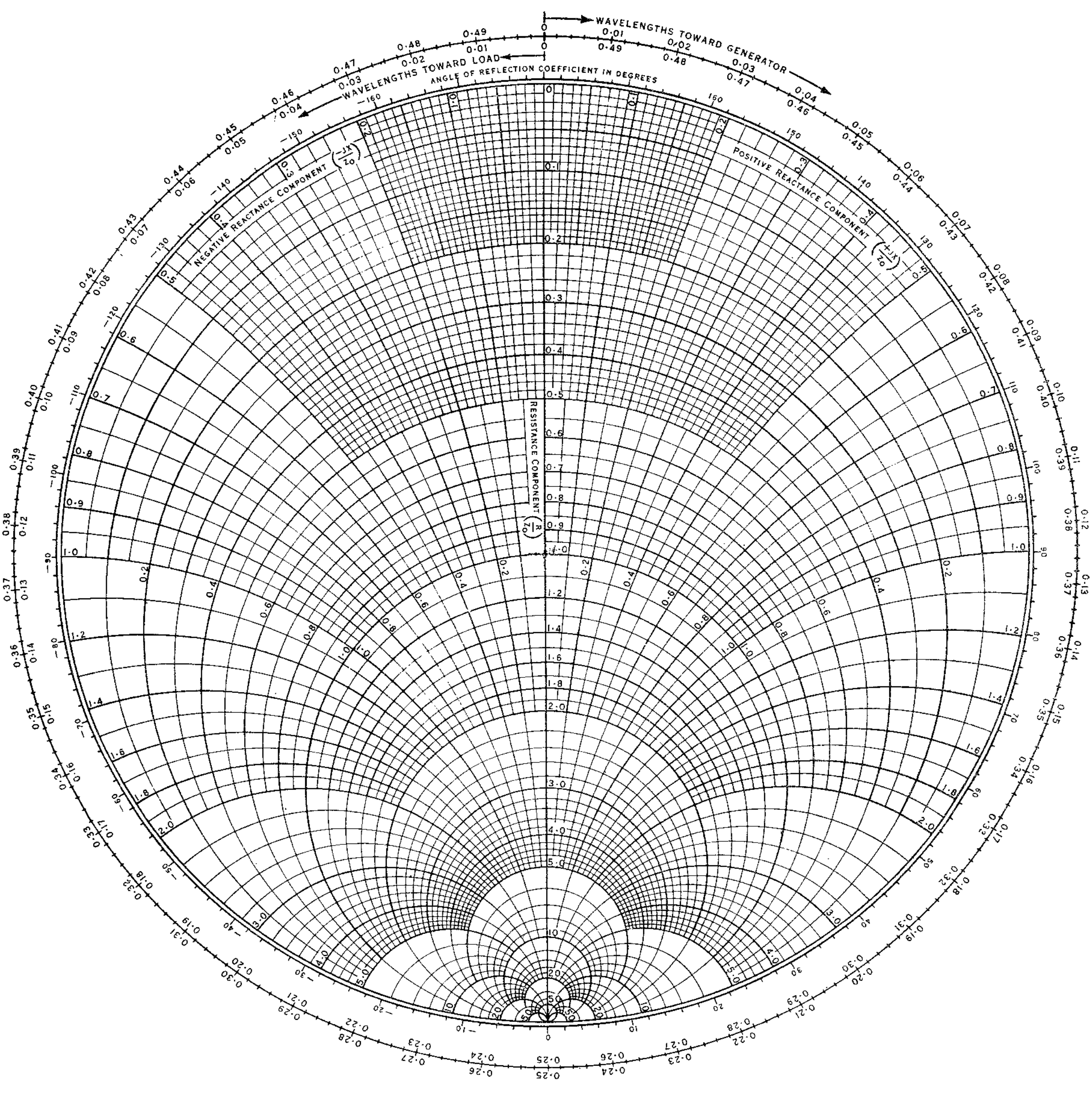
FIG. 6. DETERMINING INPUT REACTANCE OF SHORT CIRCUITED LINES.

EXERCISES

1. *Use the method shown in Fig. 6 (refer para. 4.2) to determine the input reactance in ohms of the following short circuited transmission lines. In each case, indicate whether this reactance is inductive or capacitive.*

   (a) Length = 0.176λ ; Zo = 600 ohms.
   (b) Length = 0.47λ ; Zo = 600 ohms.
   (c) Length = 0.296λ ; Zo = 50 ohms.
   (d) Length = 0.75λ ; Zo = 200 ohms.
   (e) Length = 0.9λ ; Zo = 72 ohms.

2. *What is the shortest electrical length for a short circuited transmission line (Zo = 50 ohms) which has an input capacitive reactance of 100 ohms?*



4.5 Open Circuited Lines (Fig. 7). The impedance of the open circuit termination is plotted at the bottom of the Chart. Note that the wavelength scale reads 0.25λ at this reference point, from which the length of line is measured. For example, to find the input reactance of a 600 ohm line 0.2λ long, move in a clockwise direction to the 0.45λ (that is, 0.2λ + 0.25λ) point on the wavelength scale and read the normalized value as 0 - j0.325. Therefore, the input reactance is 195 ohms and it is purely capacitive.

When this line has a length of 0.4λ, read the normalized value at 0.4λ + 0.25λ = 0.65λ on the clockwise wavelength scale. This is the same as 0.15λ, and the value is 0 + j1.38. Therefore, the input reactance is 828 ohms and it is purely inductive.

For 0.25λ, Zin is a short circuit; and for 0.5λ, Zin is an open circuit.

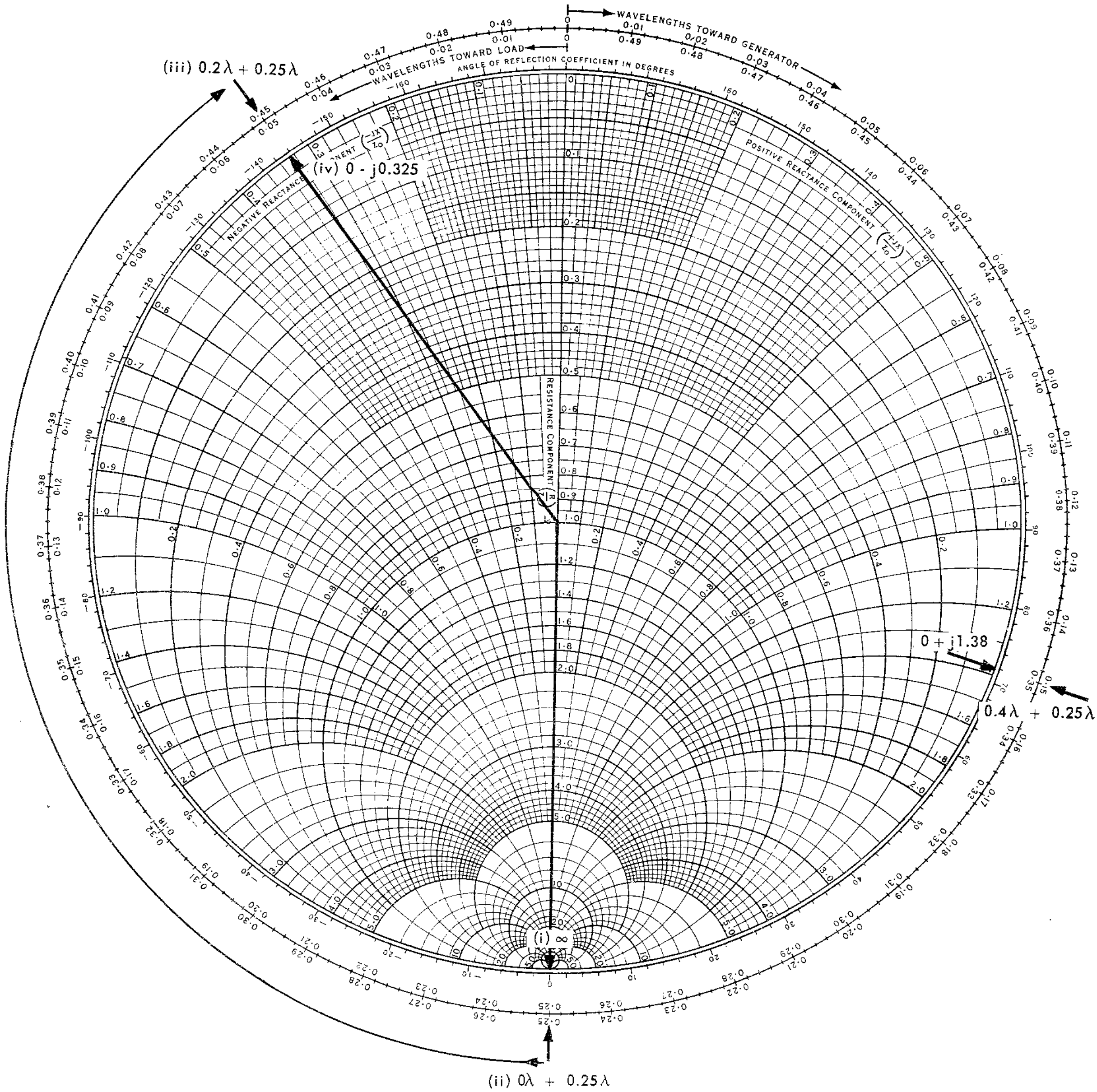


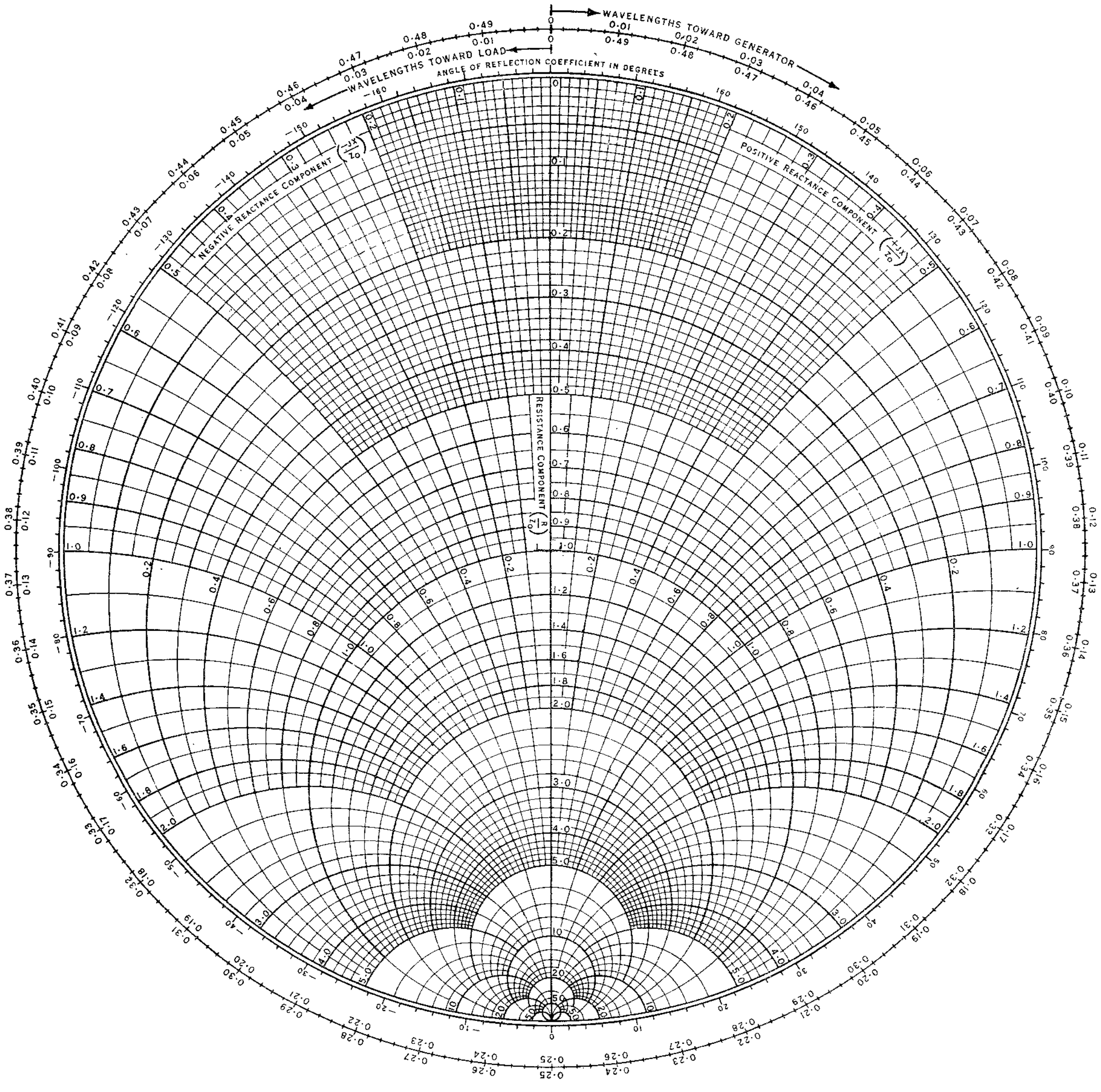
FIG. 7. DETERMINING INPUT REACTANCE OF OPEN CIRCUITED LINES.

EXERCISES

1. *Use the method shown in Fig. 7 (refer para. 4.2), to determine the input reactance in ohms of the following open circuited transmission lines. In each case, indicate whether this reactance is inductive or capacitive.*

   (a) Length = 0.176λ ; Zo = 600 ohms.
   (b) Length = 0.47λ ; Zo = 600 ohms.
   (c) Length = 0.296λ ; Zo = 50 ohms.
   (d) Length = 0.75λ ; Zo = 200 ohms.
   (e) Length = 0.9λ ; Zo = 72 ohms.

   (Compare your answers with those obtained for the similar short circuited lines referred to on page 8. What are your observations regarding the nature and magnitude of the input reactances for similar open and short circuited lines?)



4.6 Purely Reactive Terminations (Fig. 8). The normalized value of the line termination is first plotted for use as a reference point from which the length of line is measured in a clockwise direction around the "Wavelengths toward Generator" scale.

For example, assuming a 600 ohm line terminated in an inductive reactance = 240 ohms, the normalized impedance is 0 + j0.4. Note that the wavelength scale reads 0.06λ at this point. When this line has a length of -

0.10λ (that is, 0.16λ on the scale), Zin = 948 ohms (purely inductive);
0.19λ (that is, 0.25λ on the scale), Zin = an open circuit;
0.36λ (that is, 0.42λ on the scale), Zin = 330 ohms (purely capacitive);
0.44λ (that is, 0.50λ on the scale), Zin = a short circuit.

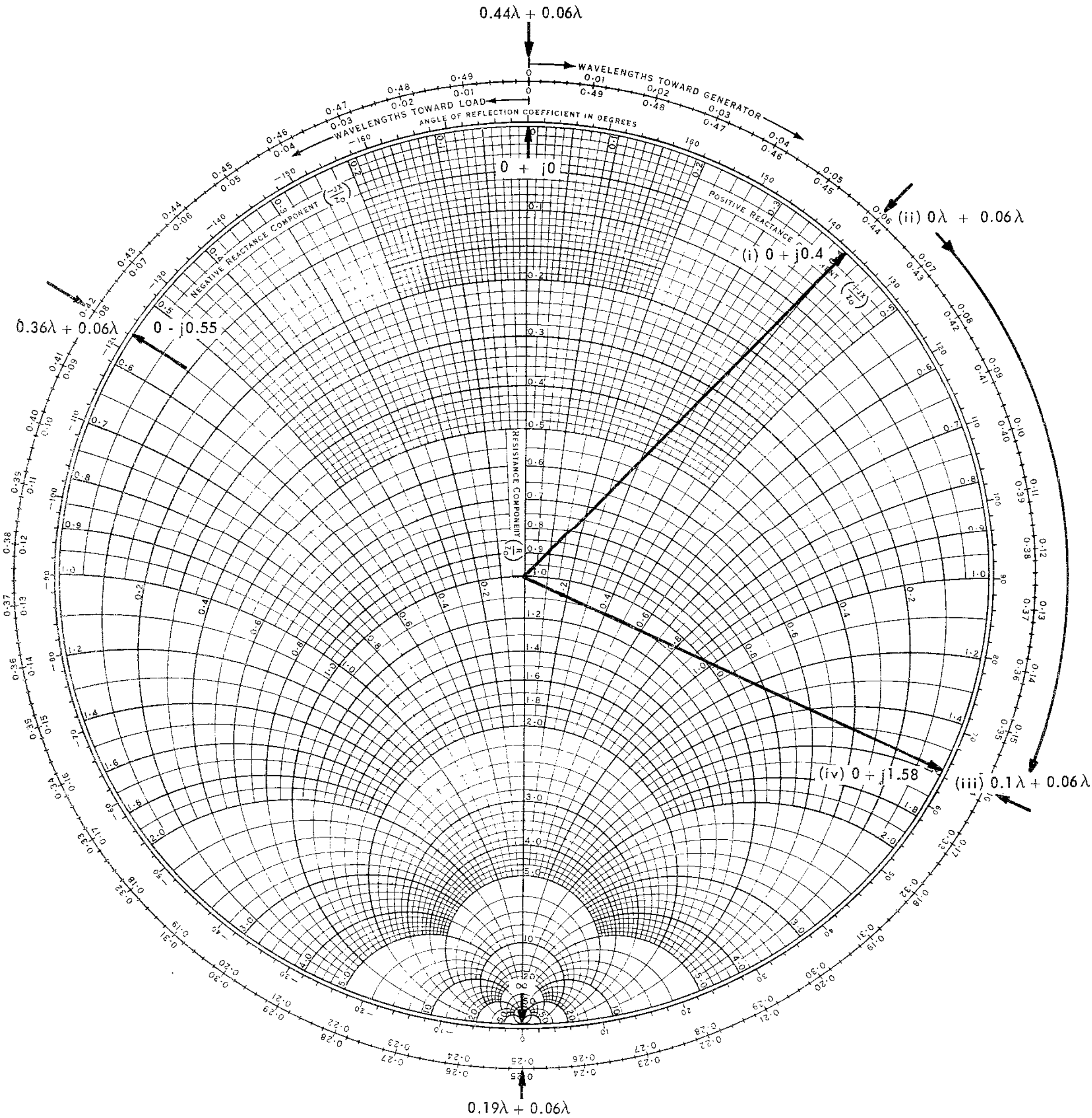
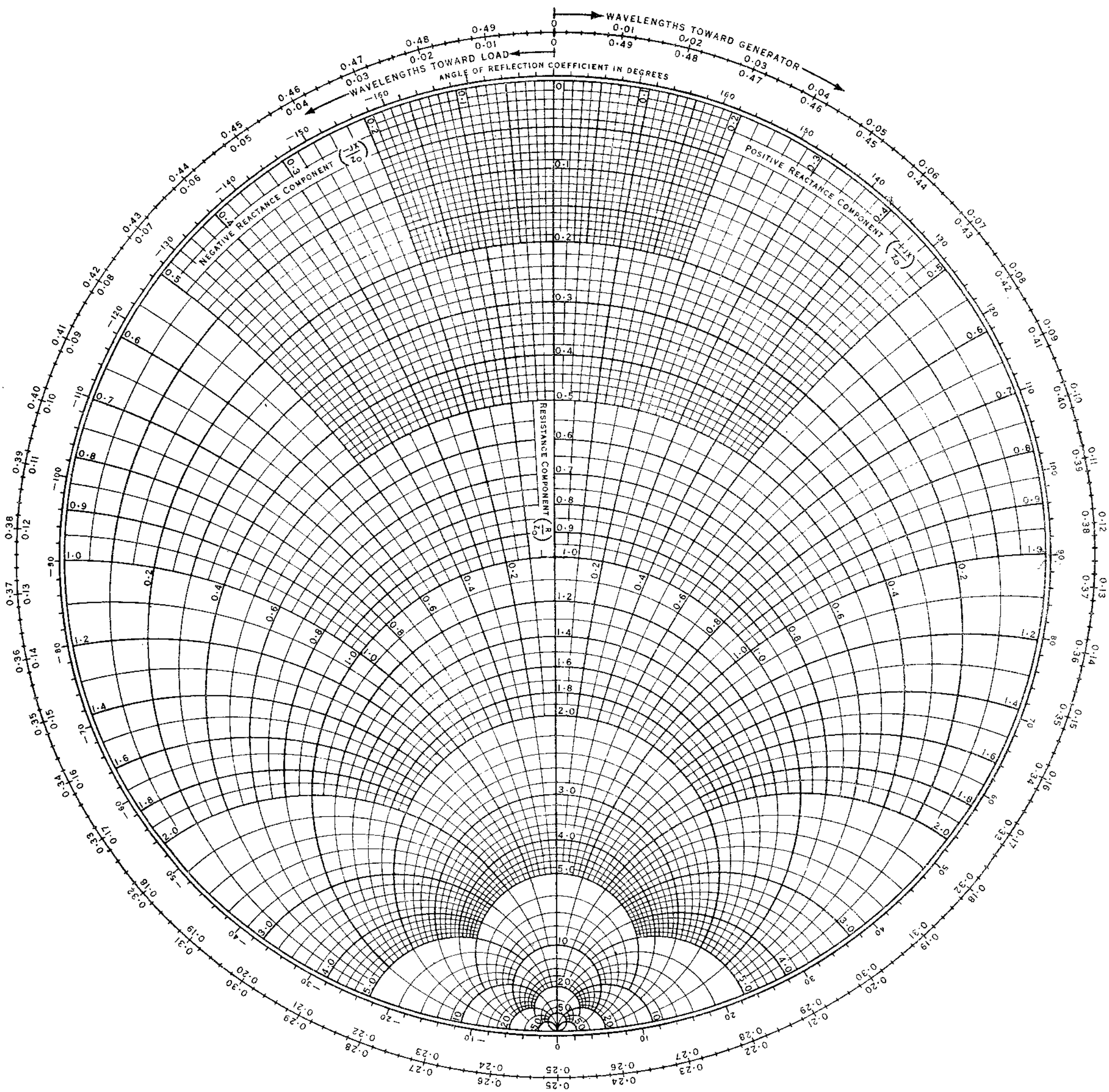


FIG. 8. DETERMINING INPUT REACTANCE OF LINES TERMINATED IN A PURE REACTANCE.

EXERCISES

1. *Use the method shown in Fig. 8 (refer para. 4.2) to determine the input reactance in ohms of the following transmission lines terminated in a pure reactance (X_T). In each case, indicate whether this reactance is inductive or capacitive.*

   (a) $X_T = 0 + j20$ ohms ; Length = 0.25λ ; Zo = 50 ohms.
   (b) $X_T = 0 - j50$ ohms ; Length = 0.1λ ; Zo = 50 ohms.
   (c) $X_T = 0 - j400$ ohms ; Length = 0.2λ ; Zo = 600 ohms.
   (d) $X_T = 0 + j330$ ohms ; Length = 0.55λ ; Zo = 300 ohms.
   (e) $X_T = 0 - j440$ ohms ; Length = 0.432λ ; Zo = 200 ohms.

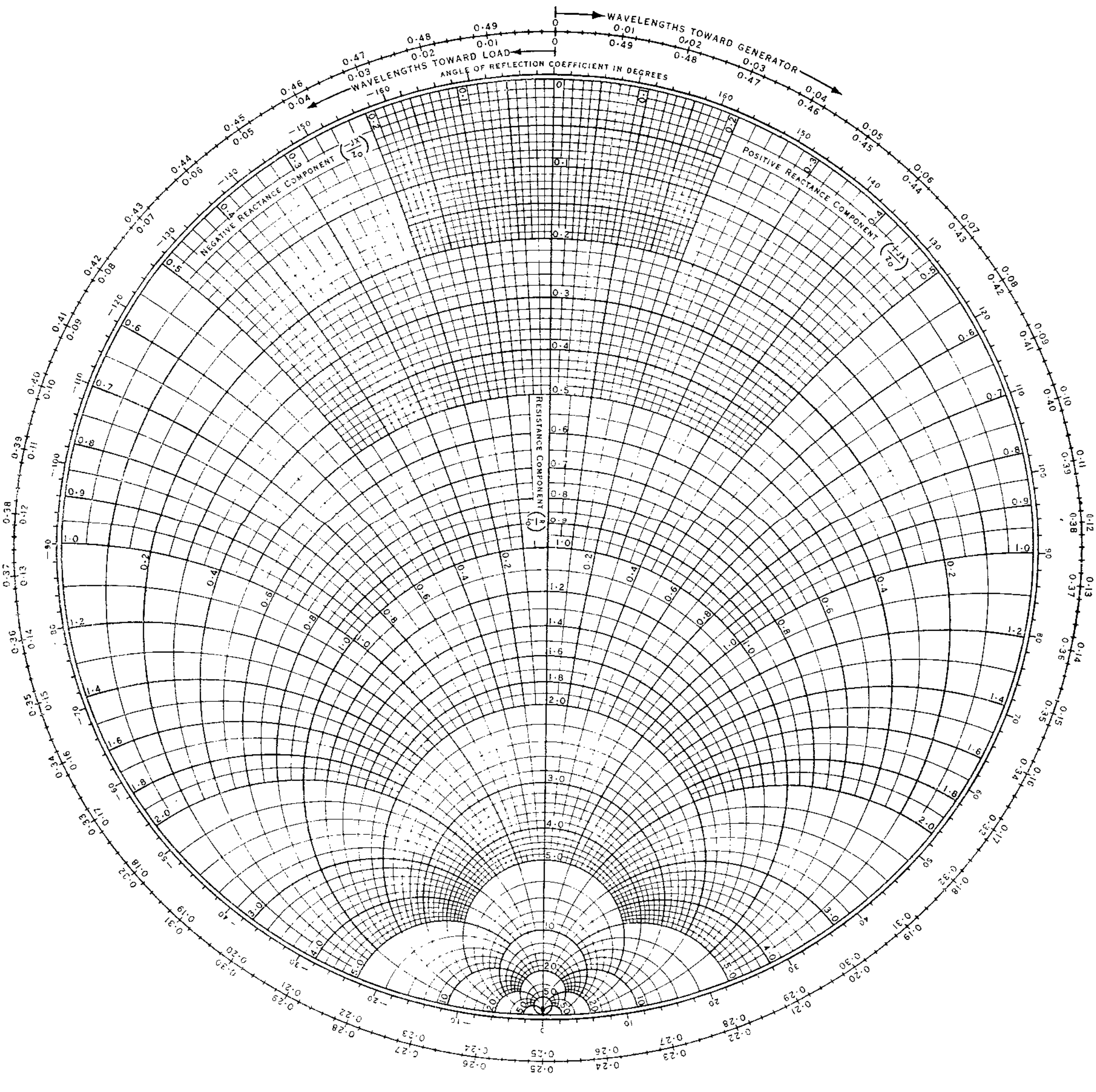


EXERCISES

1. *What is the shortest electrical length for an open circuited transmission line (Zo = 300 ohms) which has an input impedance of 0 + j75 ohms?*

2. *A transmission line (Zo = 600 ohms) is terminated in an inductive reactance of 300 ohms. What is the shortest electrical length of this line which has an input impedance equivalent to -*

   (a) an open circuit, *(b) a short circuit,* *(c) 0 - j300 ohms?*



5. STANDING WAVE RATIO CIRCLES.

5.1 Standing Wave Ratio (SWR) circles are not printed on the Chart but may be added with a drawing compass during the process of solving problems. They are centred on prime centre and appear as concentric circles inside the reactance axis. Just as the outer circle around the reactance axis gives the values of Zin for various line lengths for the extreme conditions (short circuit or open circuit), SWR circles give the values of Zin for various lengths of lines with other terminations.

An SWR circle indicates the maximum and minimum impedance variations along a line, which are related to the maximum and minimum voltage values, that is, VSWR. In practice, the VSWR value for a given circle is determined directly from the Chart, by reading the resistance value at the point where the circle crosses the resistance axis, below prime centre. (The reading where the circle crosses the resistance axis above prime centre, indicates the inverse ratio).

5.2 For example, assume that a load mismatch in a length of line causes a VSWR of 3 : 1. When line losses are negligible, the VSWR remains constant throughout the entire length of this line. This is represented on the Chart by drawing a 3 : 1 constant SWR circle (a circle with a radius of 3 on the resistance axis) as in Fig. 9. It is of interest to note that the outer circle (reactance axis) of the Chart, represents an SWR circle with a value of infinity.

5.3 SWR circles are drawn to determine the Zin of a transmission line terminated in either a pure resistance not equal to the Zo of the line or a complex impedance ($R \pm jX$). The procedure is as follows:-

(i) Normalize the load impedance and plot the normalized value on the Chart.

(ii) Draw a constant SWR circle passing through the plotted point. The standing wave ratio is indicated by the reading on the resistance axis below prime centre, at the point of intersection with the SWR circle.

(iii) Draw a straight line from prime centre through the plotted point to intersect the "Wavelengths toward Generator" scale, and read the value from the scale.

(iv) Using the value obtained in (iii) as the reference point, proceed in a clockwise direction around the wavelength scale, a distance equal to the length of the line. Draw a straight line from prime centre to this point.

(v) Read the normalized value of impedance at the point where this line intersects the SWR circle. Multiply this value by Zo to determine Zin.

5.4 A similar procedure is used to determine the load impedance when the Zin, Zo and length of line are known; except that we plot the normalized Zin as the starting point and use the anti-clockwise "Wavelengths toward Load" scale.

5.5 <u>Load Resistance less than Zo</u>. Fig. 9 shows how to use the Chart to find the Zin of a 600 ohm line 0.35λ long, which is terminated in a load resistance of 200 ohms. Using the steps listed in para. 5.3 -

(i) Plot the normalized value of load resistance ($0.33 + j0$) on the Chart.

(ii) Draw a constant SWR circle through this plotted point and note that the VSWR reading is 3 : 1.

(iii) Draw a straight line from prime centre through the plotted point, to intersect the "Wavelengths toward Generator" scale at 0λ.

(iv) From 0λ, proceed around the clockwise wavelength scale a distance of 0.35λ, and draw a straight line from prime centre to this point.

(v) This straight line intersects the SWR circle at $0.8 - j1.0$. Multiplying this normalized value by 600 ohms, gives a Zin of $480 - j600$ ohms for the line.

Similarly, when a 600 ohm line 0.25λ long is terminated in a load resistance of 200 ohms (one third of Zo), the value of Zin is 1800 ohms (three times Zo) and it is purely resistive. A line which is an odd number of quarter wavelengths long, inverts the load.

Conversely, when the length of this line is increased to 0.5λ, the input resistance is the same as the load resistance (200 ohms). A line which is an even number of quarter wavelengths long, repeats the load.

For lengths greater than 0.5λ, subtract a whole number of half wavelengths from the length to give a value between 0λ and 0.5λ. Thus, for 0.85λ, subtract 0.5λ and read the value of input impedance as for 0.35λ.

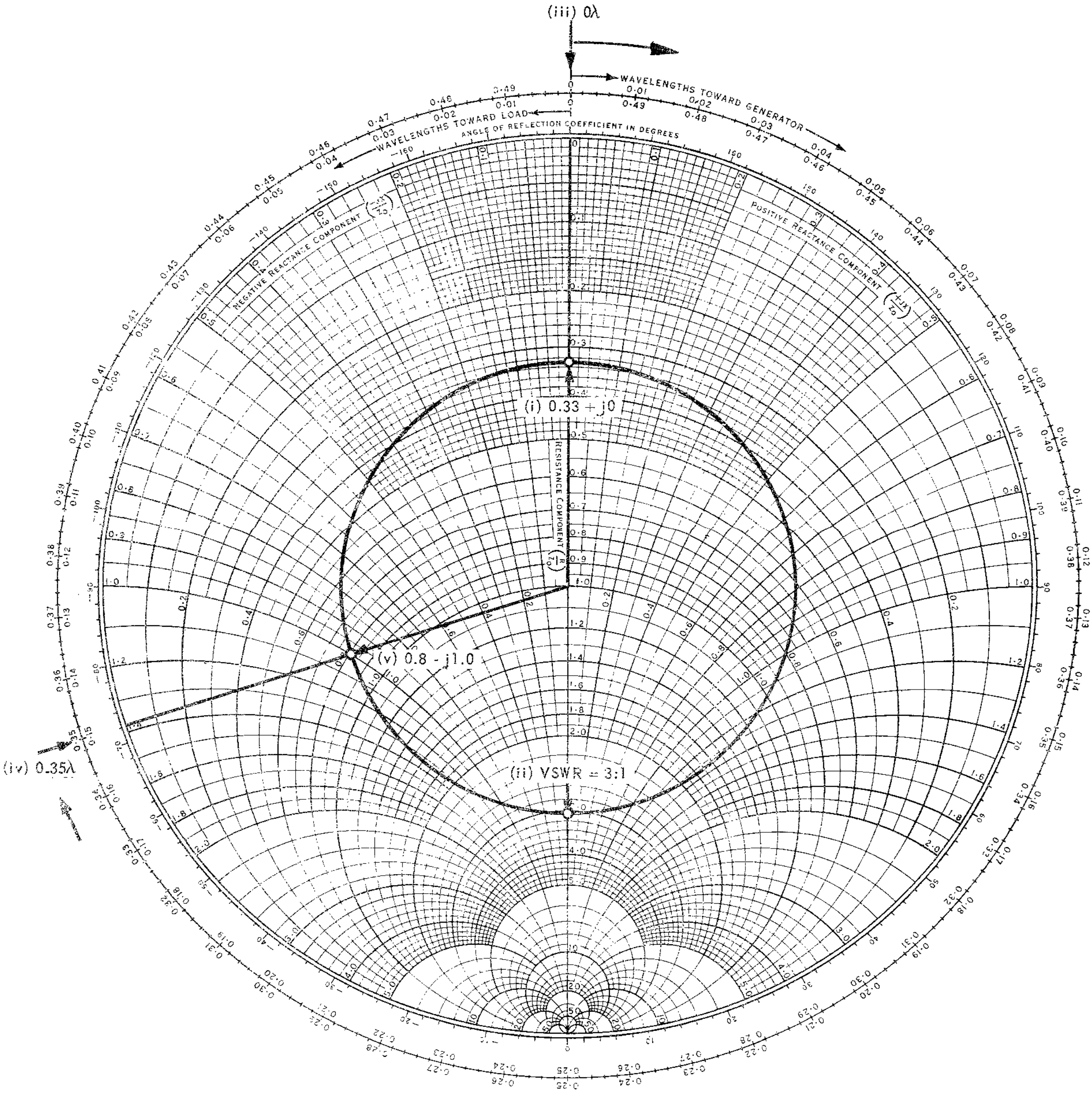
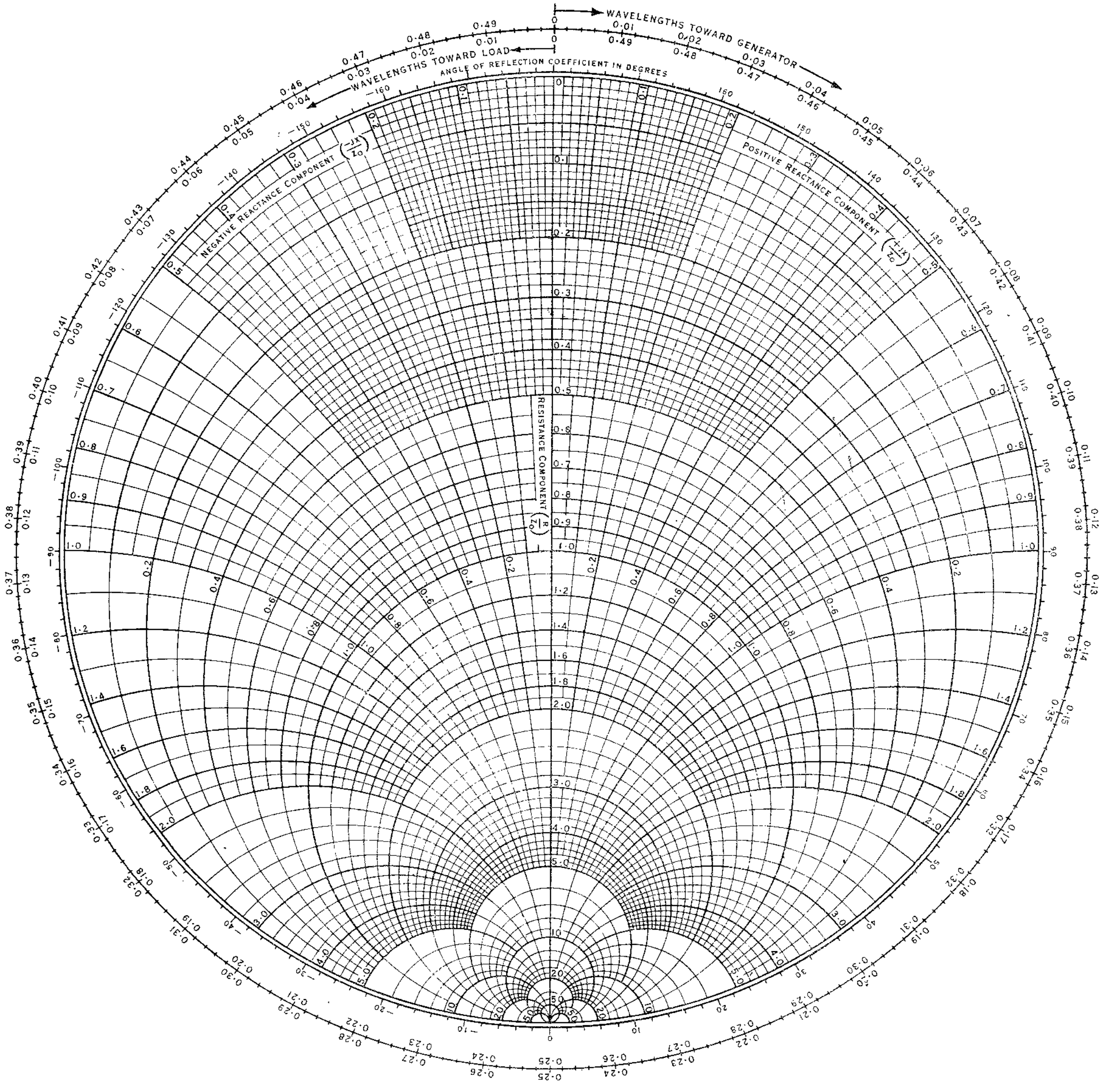


FIG. 9. DETERMINING Zin OF LINES TERMINATED IN PURE RESISTANCE LESS THAN Zo.

EXERCISES

1. *Use the method shown in Fig. 9 (refer para. 5.3) to determine the Zin of the following transmission lines terminated in a pure resistance (R_T) less than the Zo of the line. In each case, express Zin in the form R ± jX.*

(a) R_T = 100 ohms; Length = 0.35λ ; Zo = 300 ohms.
(b) R_T = 200 ohms; Length = 0.605λ ; Zo = 600 ohms.
(c) R_T = 10 ohms; Length = 0.1λ ; Zo = 50 ohms.
(d) R_T = 40 ohms; Length = 0.25λ ; Zo = 200 ohms.
(e) R_T = 120 ohms; Length = 0.6λ ; Zo = 600 ohms.



5.6 Load Resistance greater than Zo. Fig. 10 refers to a 600 ohm transmission line 0.35λ long and terminated in a load resistance of 1,800 ohms. Note that the straight line drawn from prime centre through the plotted point (3 + j0) corresponding to the normalized value of load resistance, intersects the "Wavelengths toward Generator" scale at 0.25λ, from which the length of line is measured. This line has a VSWR of 3 : 1, a normalized Zin of 0.49 + j0.62, and an actual Zin of 294 + j372 ohms.

When this line is 0.25λ long, Zin = 200 ohms and is purely resistive; and for 0.5λ, Zin = 1,800 ohms and is purely resistive.

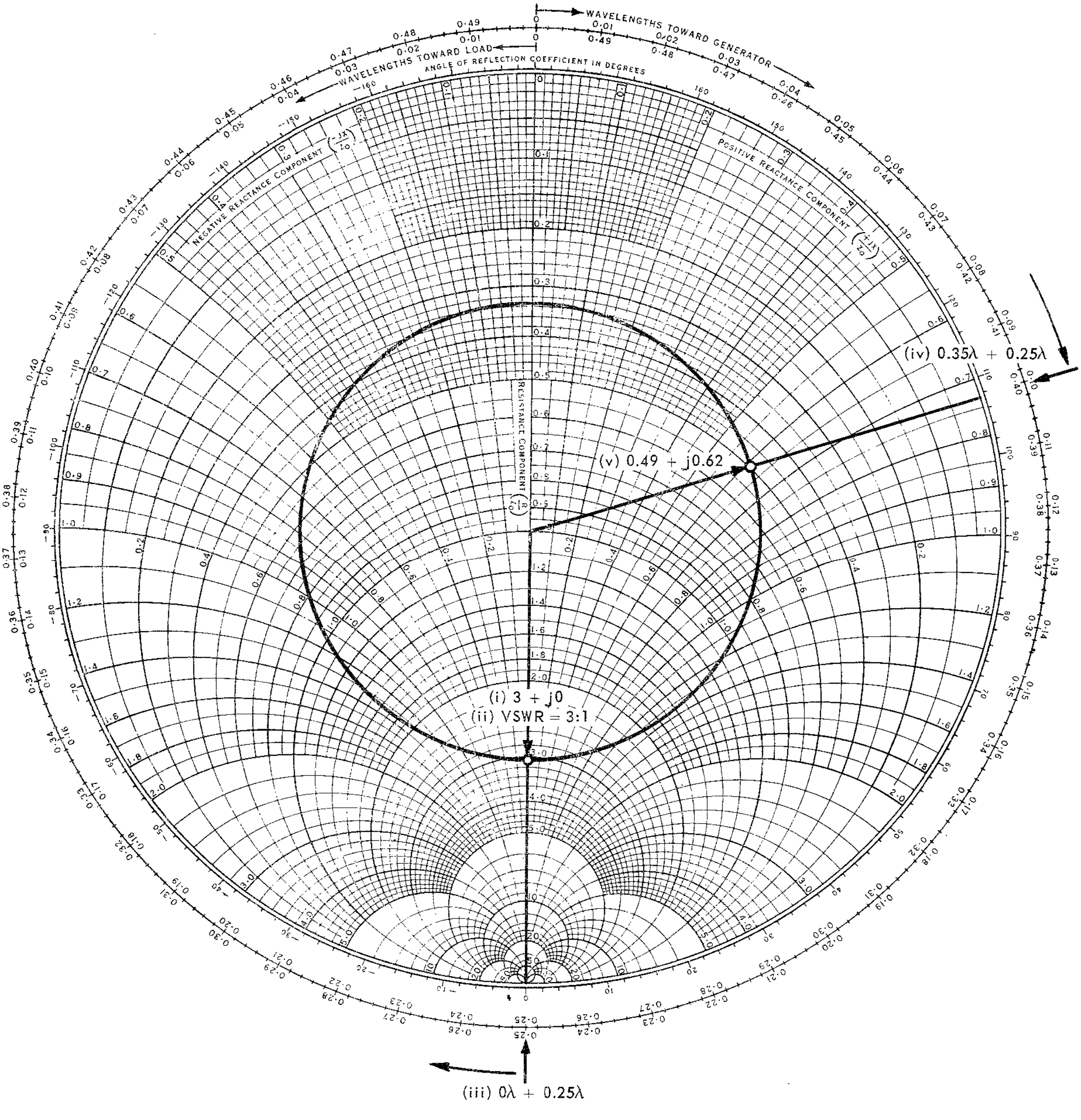
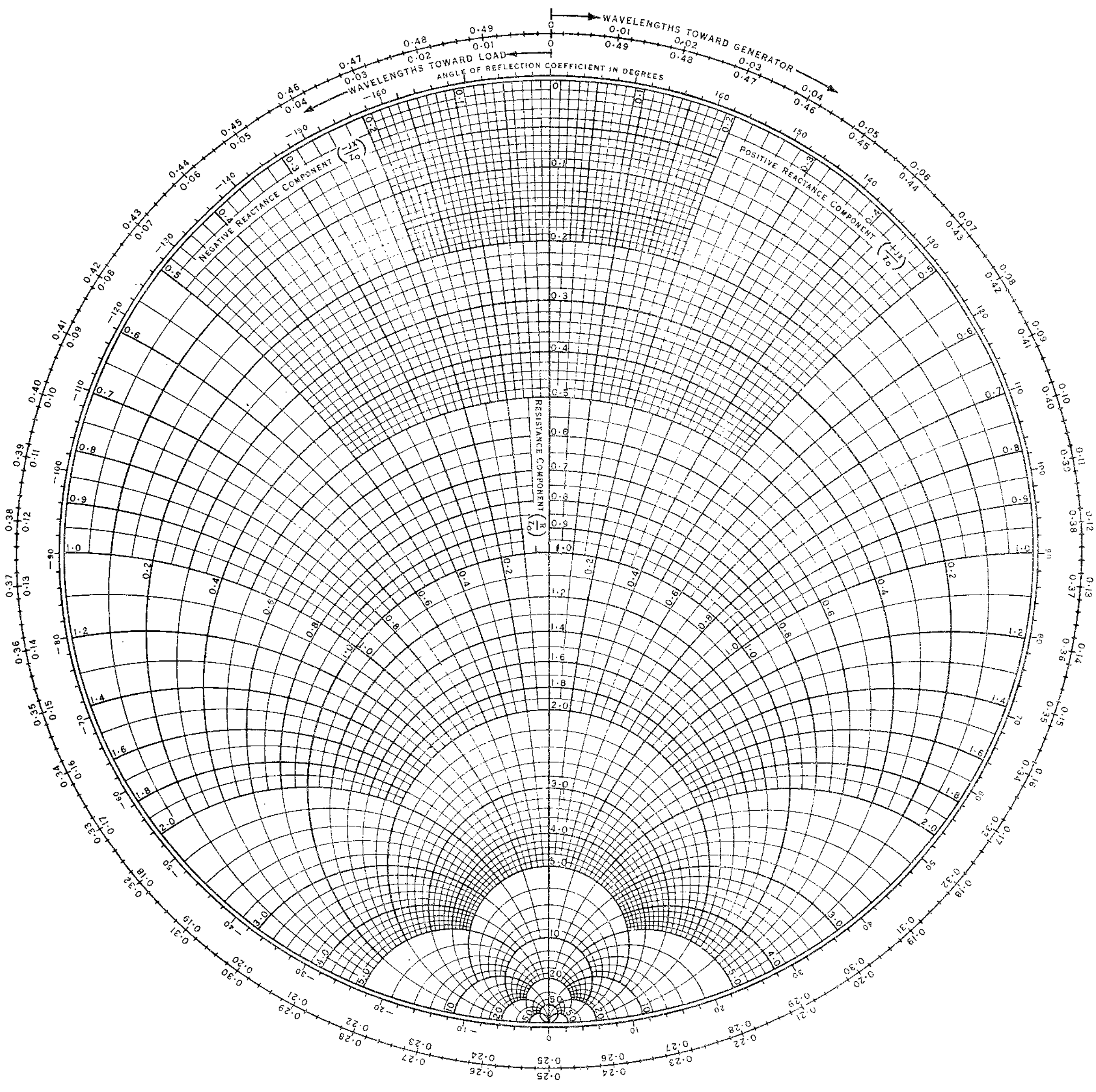


FIG. 10. DETERMINING Zin OF LINES TERMINATED IN PURE RESISTANCE GREATER THAN Zo.

EXERCISES

1. *Use the method shown in Fig. 10 (refer para. 5.3) to determine the Zin of the following transmission lines terminated in a pure resistance (R_T) greater than the Zo of the line. In each case, express Zin in the form R ± jX.*

(a) R_T = 600 ohms. Length = 0.16λ ; Zo = 200 ohms.
(b) R_T = 300 ohms. Length = 0.09λ ; Zo = 100 ohms.
(c) R_T = 400 ohms. Length = 0.39λ ; Zo = 200 ohms.
(d) R_T = 600 ohms. Length = 1.1λ ; Zo = 300 ohms.
(e) R_T = 840 ohms. Length = 0.25λ ; Zo = 200 ohms.



5.7 Line Terminated in Complex Impedance. Fig. 11 applies to a 50 ohm transmission line terminated in a complex impedance of 25 + j25 ohms. The SWR circle indicates that this line has a VSWR of 2.67 : 1. For a length of 0.3λ, the normalized Zin is 0.6 - j0.65 and the actual Zin is 30 - j32.5 ohms.

From this Chart, we can determine that the Zin is purely resistive and equal to 50 × 2.67 = 133.5 ohms when the length is 0.162λ (that is, 0.25λ on the scale).

Also, Zin is purely resistive and equal to 50 × 0.375 = 18.75 ohms when the length is 0.412λ (that is, 0.5λ on the scale).

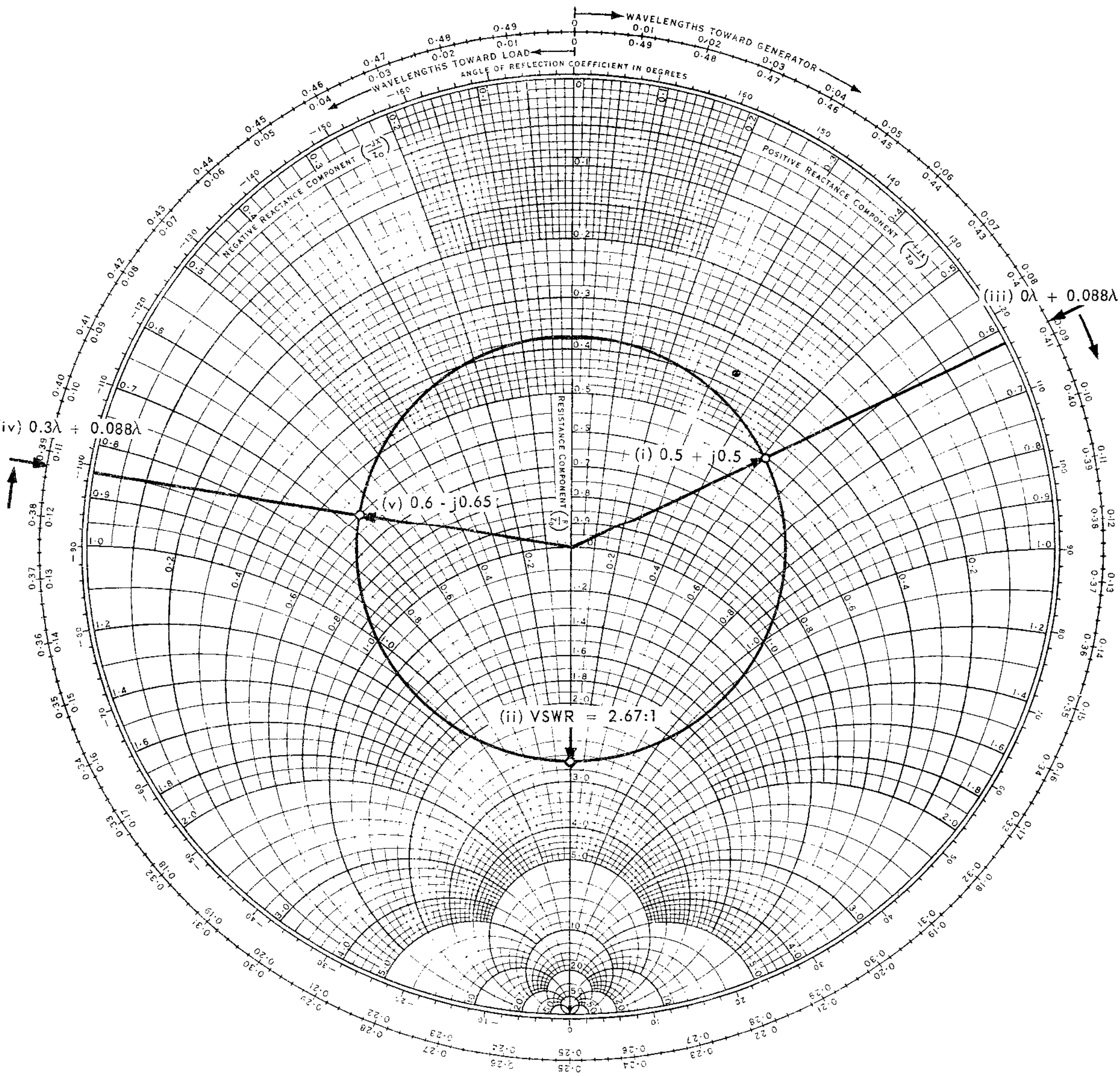
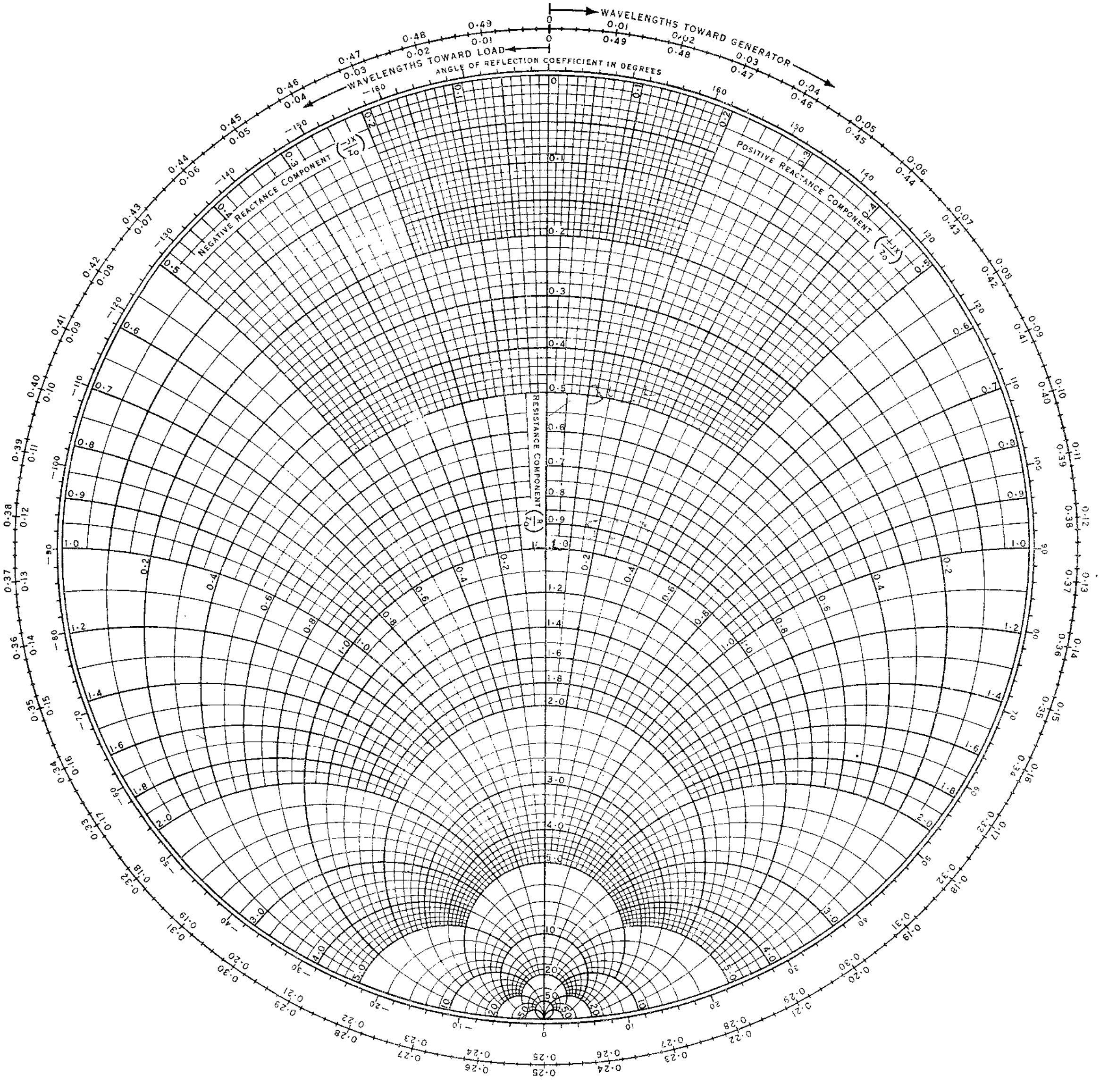


FIG. 11. DETERMINING Zin OF LINES TERMINATED IN COMPLEX IMPEDANCE.

EXERCISES

1. *Use the method shown in Fig. 11 (refer para. 5.3) to determine the Zin (in the form R ± jX) and VSWR of each of the following lines terminated in a complex impedance (Z_T).*

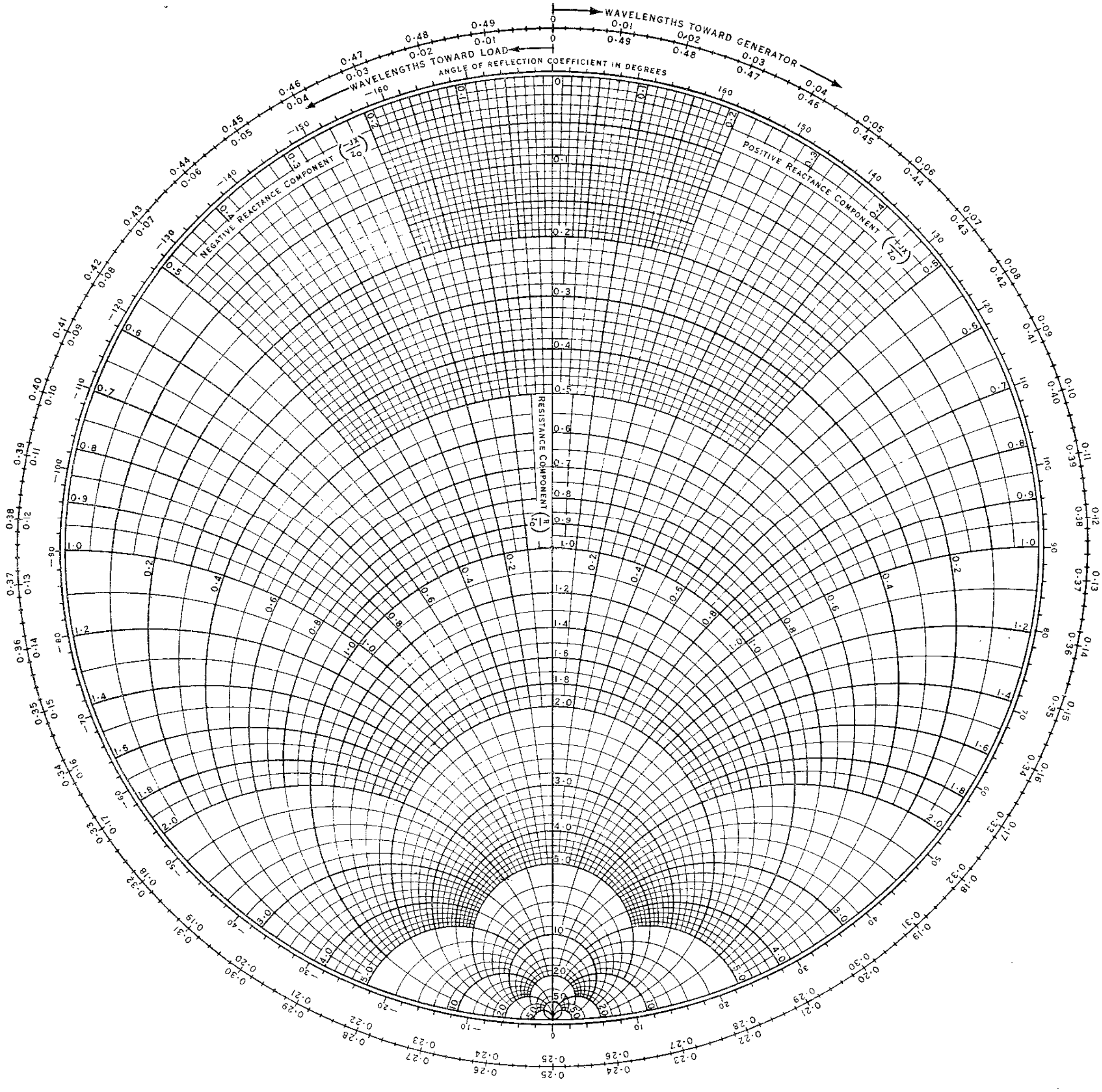
(a) Z_T = 300 - j300 ohms ; Length = 0.338λ ; Zo = 600 ohms.
(b) Z_T = 40 + j100 ohms ; Length = 0.25λ ; Zo = 100 ohms.
(c) Z_T = 240 - j600 ohms ; Length = 0.25λ ; Zo = 600 ohms.



EXERCISES

1. *The input impedance of a 300 ohm transmission line which is 2.15λ long at the applied frequency, is measured at 500 - j600 ohms. Determine:-*

   (a) the VSWR on the line;
   (b) the value of load impedance (in the form R ± jX).



6. REFLECTION COEFFICIENT SCALES.

6.1 In addition to the line input impedance (or load impedance) and the VSWR, the Chart reveals several other operating characteristics of a transmission line system. For example, the angle of reflection coefficient for a particular load is given. This indicates the angle by which the reflected voltage lags the incident voltage wave at the load. The value in degrees is read from the line drawn from prime centre through the plot of the load impedance where the line intersects the "Angle of Reflection Coefficient" scale, which is included just inside the wavelengths scales. Note that angles on the left half, or capacitive reactance side, of the Chart are negative angles, a "negative lag" indicating that the reflected voltage wave leads the incident wave.

For short circuited lines (Fig. 6) or lines terminated in a load resistance less than Zo (Fig. 9), the angle of reflection coefficient is 180°, that is, the reflected voltage is 180° out of phase with the incident voltage at the load.

For open circuited lines (Fig. 7) or lines terminated in a load resistance greater than Zo (Fig. 10), the angle of reflection coefficient is 0°, that is, the reflected voltage is in phase with the incident voltage at the load.

For purely inductive loads or for lines terminated in a complex impedance which is predominantly inductive, the reflected voltage lags the incident voltage by an angle between 0° and 180°. For example, in Fig. 8, this angle of lag is about 136.7°; in Fig. 11, it is about 116.5°.

Conversely, for purely capacitive loads or for lines terminated in a complex impedance which is predominantly capacitive, the reflected voltage leads the incident voltage by an angle between 0° and 180°, and is read on the left hand side of the Chart.

6.2 Also, on some versions of the Chart, the values of reflection coefficient, reflection loss, and the dB equivalent of the VSWR on the line, etc., may be read from either external scales at the side of the Chart or a cursor pivoted at prime centre.

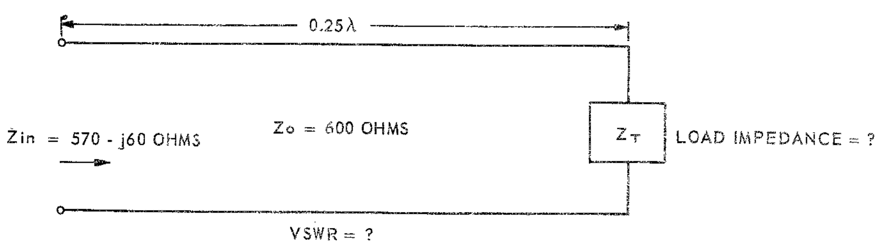
7. EXPANDED SMITH CHART.

7.1 Fig. 12 shows an expansion of the centre portion of the Smith Chart which is used for greater accuracy with small values of VSWR. The use of this Chart is similar to that shown in previous examples.

7.2 Example. The Zin of a 600 ohm transmission line terminated in an impedance (Z_T) is measured at 570 - j60 ohms. Assuming the length of the line is 0.25λ determine -

(a) the value of load impedance;

(b) the VSWR on the line.



Solution. *The procedure is similar to that outlined in para. 5.4. The results of the steps are plotted in Fig. 12.*

(i) The normalized Zin is 0.95 - j0.1.
(ii) The constant SWR circle indicates that the VSWR is 1.118 : 1.
(iii) The straight line from prime centre through the plotted point intersects the "Wavelengths toward Load" scale at 0.092λ.
(iv) From this point, proceed in an anti-clockwise direction, a distance 0.25λ around the scale. (This corresponds to a scale reading of 0.342λ).
(v) The straight line from prime centre to this point intersects the SWR circle at the normalized impedance value of 1.04 + j0.11, which gives a load impedance of 624 + j66 ohms.

Answers: (a) *624 + j66 ohms;* (b) *VSWR = 1.118 : 1.*

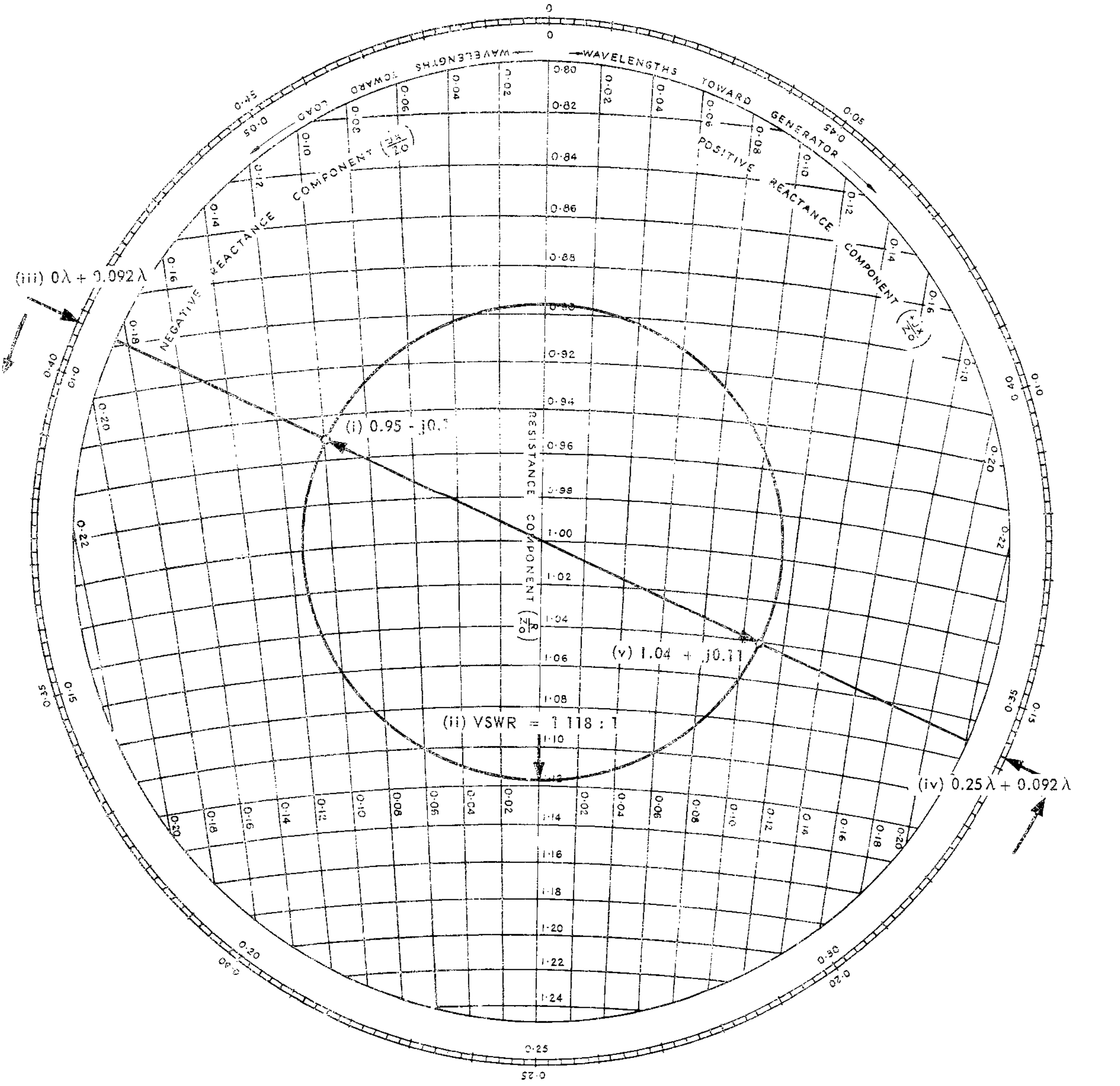
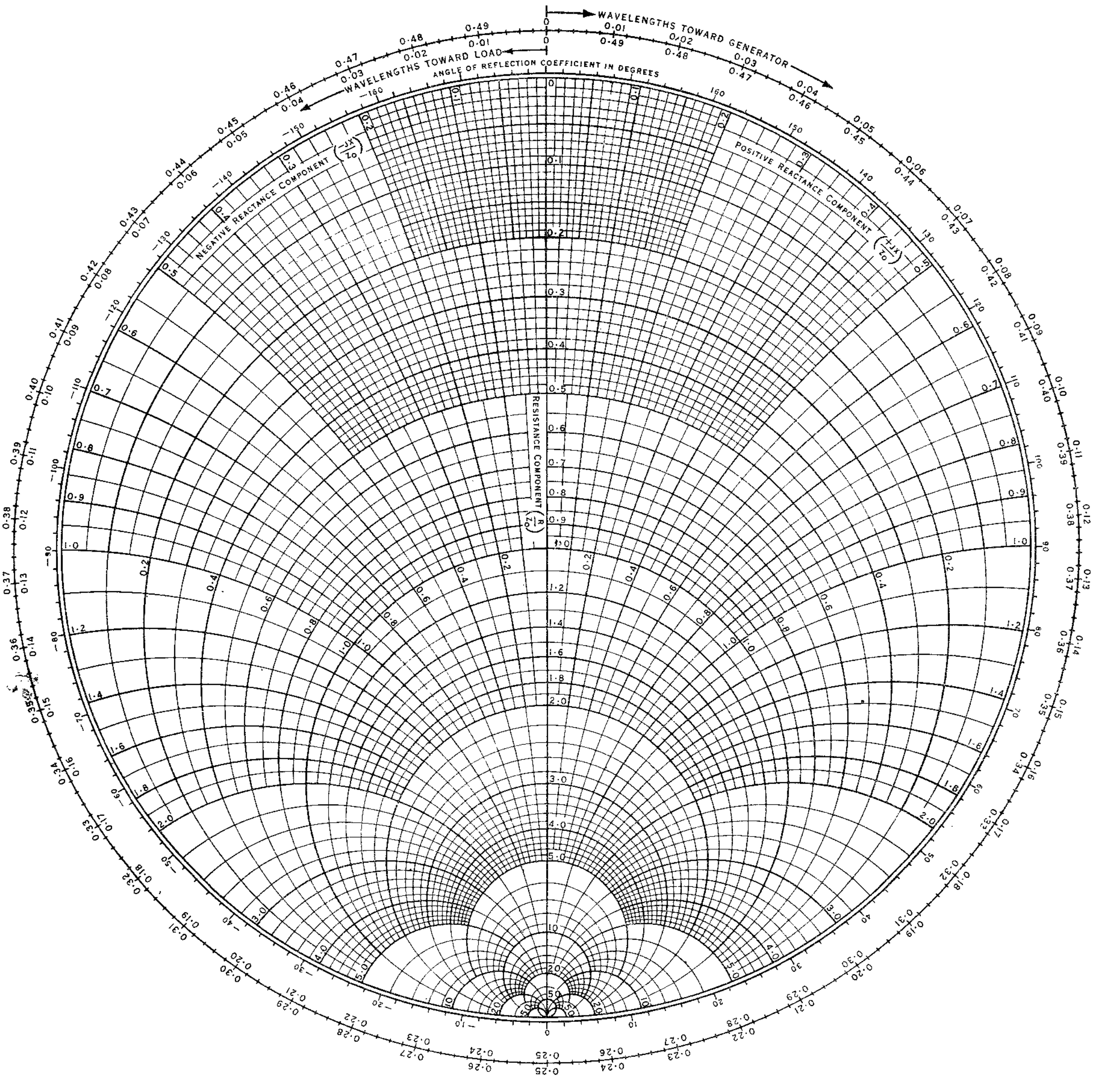


FIG. 12. USING THE EXPANDED SMITH CHART.

8. *TEST QUESTIONS.*

1. *A 600 ohm transmission line is terminated in a load impedance of $200\angle 0^\circ$ ohms. What is the shortest electrical length (in wavelengths) of this line which has a $Z_{in} = R - jX$ where $R = 300$ ohms?*

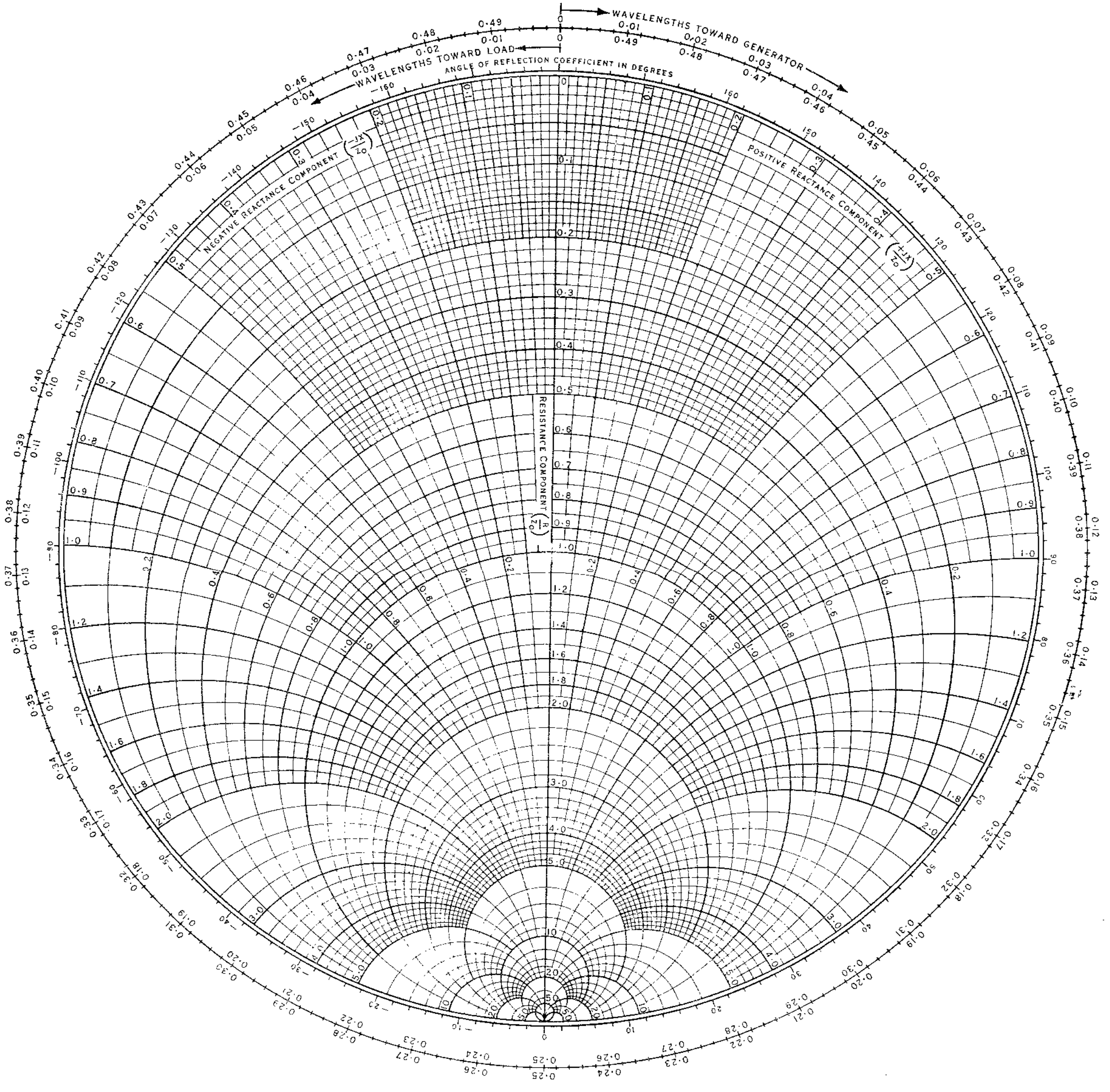


2. *When a 600 ohm transmission line is terminated in a load resistance, a VSWR of 2 : 1 exists on the line and a current node occurs at the load.*

Determine -

(a) the line impedance at a point 0.2λ back from the termination;

(b) the angle of reflection coefficient of voltage at the load.



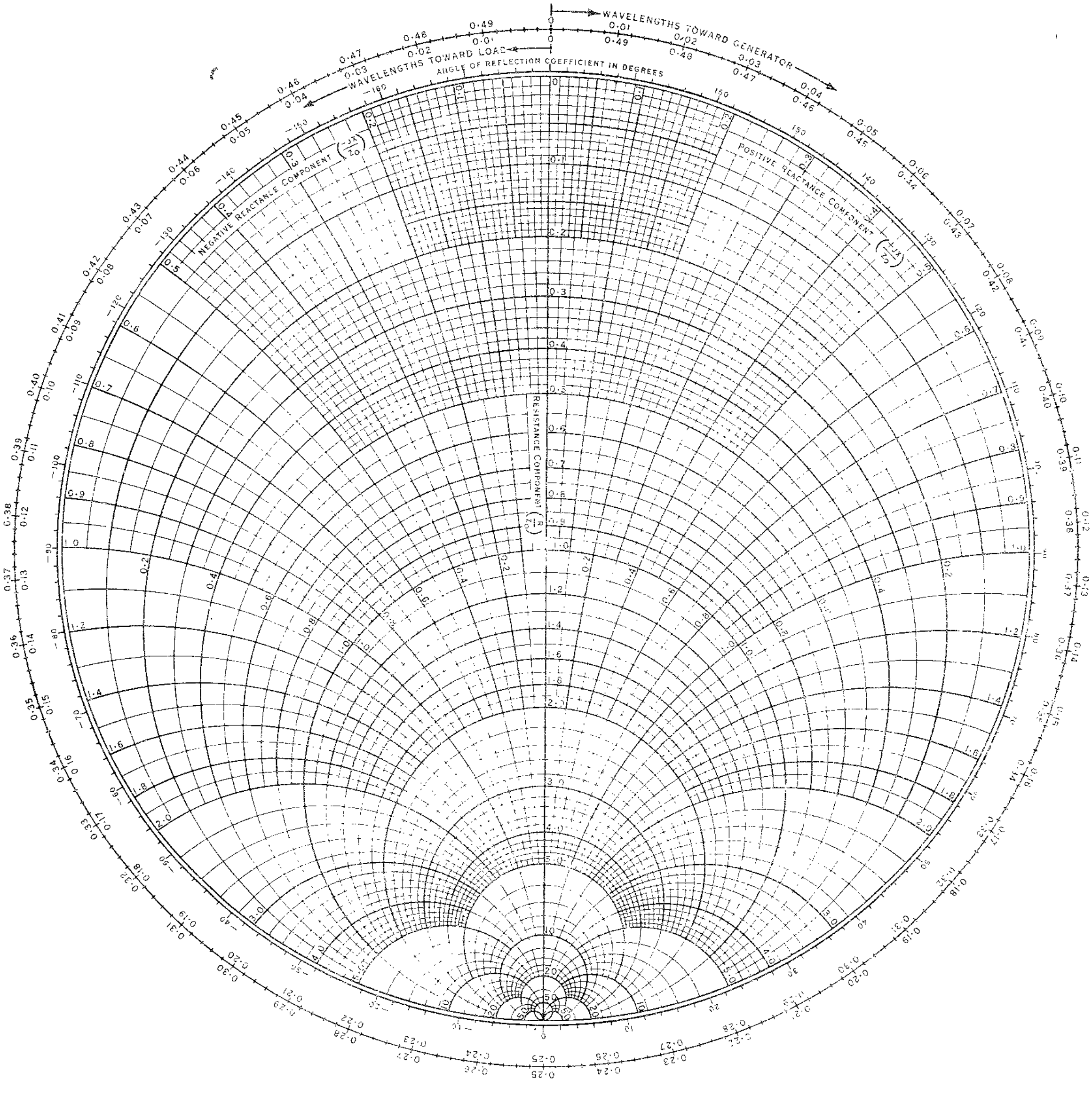
3. *A 50 ohm transmission line is terminated in an impedance of 25 - j25 ohms.*

Determine -

(a) the VSWR on this line;

(b) the angle of reflection coefficient of voltage at the load;

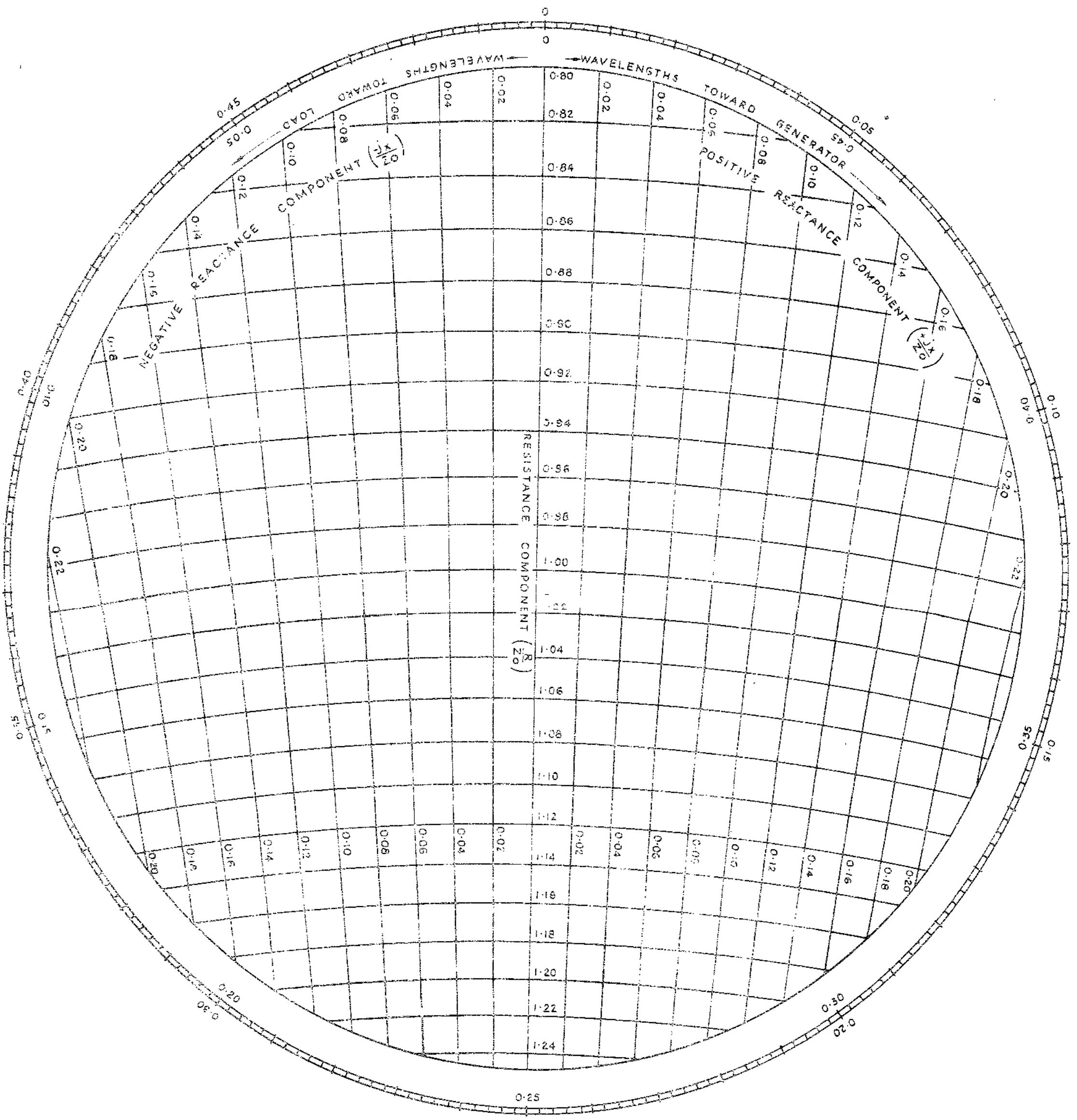
(c) the shortest distance (in wavelengths) back from the termination to the point at which the line impedance is equivalent to a resistance of 25 ohms in series with an inductive reactance.



4. *A 300 ohm transmission with negligible attenuation is connected to a load with an impedance of 330 + j30 ohms. Assuming the electrical length of the line is 1¼λ, calculate -*

   (a) the VSWR on the line;

   (b) the input impedance of the line.



ANSWERS

Page 5.

2. (a) 20 - j50 ohms.
   (b) 40 + j600 ohms.
   (c) 90 + j630 ohms.
   (d) 0 + j0 ohms.

Page 8.

1. (a) X_L = 1,200 ohms.
   (b) X_C = 114 ohms.
   (c) X_C = 170 ohms.
   (d) Open Circuit.
   (e) X_C = 52 ohms.

2. 0.324λ.

Page 10.

1. (a) X_C = 300 ohms.
   (b) X_L = 3,160 ohms.
   (c) X_L = 15 ohms.
   (d) Short Circuit.
   (e) X_L = 100 ohms.

Page 12.

1. (a) X_C = 125 ohms.
   (b) X_C = 8 ohms.
   (c) X_L = 480 ohms.
   (d) X_L = 660 ohms.
   (e) Open Circuit.

Page 13.

1. (a) 0.289λ.

2. (a) 0.177λ.
   (b) 0.427λ.
   (c) 0.354λ.

Page 16.

1. (a) 240 - j300 ohms.
   (b) 315 + j390 ohms.
   (c) 15 + j35 ohms.
   (d) 1,000 + j0 ohms.
   (e) 180 + j420 ohms.

Page 18.

1. (a) 92 - j110 ohms.
   (b) 92 - j110 ohms.
   (c) 180 + j135 ohms.
   (d) 290 - j210 ohms.
   (e) 47.6 + j0 ohms.

Page 20.

1. (a) 1,560 + j0 ohms.
   VSWR = 2.67 : 1.
   (b) 35 - j87.5 ohms.
   VSWR = 5.3 : 1.
   (c) 210 - j525 ohms.
   VSWR = 5.3 : 1.

Page 21.

1. (a) 4.6 : 1.
   (b) 175 + j350 ohms.

Pages 24 - 27.

1. 0.39λ.
2. (a) 330 - j150 ohms.
   (b) 0°.
3. (a) VSWR = 2.67 : 1.
   (b) 116.5° leading.
   (c) 0.176λ.
4. (a) VSWR = 1.142 : 1.
   (b) 270 - j24 ohms.

END OF PAPER.